Socio-economic impact of mathematical research and mathematical technology in Spain

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Red Estratégica en Matemáticas

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Spanish State Research Agency

The Red Estratégica en Matemáticas (REM) was set up in 2016 as a network of all relevant nodes in research and mathematical transfer in Spain, integrating in fact the entire research community in this field. It started based upon the high structuring of this community, begun decades ago.

The REM is a project funded by the Spanish National Research Agency, and is formed by

CRM/BGSMATH (Center for Mathematical Research/Barcelona Graduate School in Mathematics),
BCAM (Basque Center for Applied Mathematics),
ICMAT (Institute of Mathematical Sciences),
RedIUM (Network of University Institutes of Mathematics),
MATH-IN (Spanish Network Mathematics-Industry),
Public High Education Centers,

coordinated by the IMUS (Institute of Mathematics of the University of Seville).

Starting from the consideration of the strategic nature of mathematics recognized in the national R+D+I plans, its objectives consist of the improvement of national and international strategic positioning and the scientific and economic impact of Spanish mathematics. In this framework, six strategic actions are developed,

Action 1: Study of the results of research in mathematics in the last decade.
Action 2: Support for transversal actions of Mathematical Technology Transfer.
Action 3: Dissemination of the activity and results of mathematical research in Spain,
Action 4: Enhancement of the presence of Spanish mathematics in the world.
Action 5: Promoting the funding for Spanish mathematics.
Action 6: Study of the socio-economic impact of research and transfer of mathematical technology in Spain.

Actions from 2 to 5 directly drive the strategic positioning of mathematics in Spain in different areas. In their turn, the objective of Actions 1 and 6 is to carry out an exhaustive balance of the current situation of the scientific and economic impact of mathematics in Spain, as a basis for its subsequent enhancement.

The present study is the one foreseen in Action 6, carried out by Analistas Financieros Internacionales (AFI).

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PREFACE

Mathematics are silently helping to shape the current technological world. Mathematics not only provide a deep insight into processes and systems, and helps to improve scientific knowledge, but also contributes to generating added value in virtually all economic sectors. In addition, there has been a paradigm shift in the applications of mathematics in recent years, as they also provide direct added value to emerging sectors related to data analysis. Without research, training and mathematical transfer, there would be no engineering or economics in the way we know them today, there would be no information technology, no smartphones, no computers, no online bank accounts, no PIN numbers...

The public institutions are well aware of the strategic role of mathematics in economic and social development. In 2016, the European Commission launched a public consultation on mathematics in the H2020 Framework Program, as a basis for the future Horizon 2020 Work Program (2018-20) with innovative mathematical content. Among other considerations, it states: "*Two facts motivated this request: today's digital society depends on mathematics and algorithms; there is a vast pool of mathematical talent in Europe. The conclusion is that Europe can be first* *in mathematical applications for big data, computing and especially High Performance Computing, to be first in modern science and innovation*". In the Spanish R+D+i 2004-2007 Plan, it was stated that "*Mathematics should be considered as a key piece in the R+D+ i system of any modern society, and in particular, of the Spanish, its strategic nature must be clearly pointed out* ".

Based on these considerations, in 2016 the *Spanish State Research Agency* funded the creation of the *Red Estratégica en Matemáticas* (REM), which promotes the national and international strategic positioning of research and mathematical transfer in Spain. An output of the action of the REM is the present report on the impact of mathematics on the socio-economic development of Spanish society.

This independent report, conducted by *Analistas Financieros Internacionales* and referred to 2016, provides very relevant conclusions on how mathematics drives the Spanish economy. The impact of these in terms of economic value and employment reaches surprising values, showing the high capacity of mathematics to provide added value in a wide variety of economic sectors. Of course, mathematics is fundamental in modern digital services, communications and internet, but also in more traditional sectors such as banking, insurance, electricity and gas, among others, in which its use is experiencing a rapid development. The report also shows the high productivity of the economic branches with high intensity of use of mathematics, comparable to that of other European countries such as France, Holland or England.

It is a great satisfaction for the REM to place this report at the service of Spanish society, especially its productive sectors and its administration. We hope that it will help increase the capacity of mathematics to provide added value to the Spanish economy and improve employment and, ultimately, to contribute to improve the quality of life of its citizens.

Sevilla, April 10 2019

Tomás Chacón Rebollo
Coordinator of the
Red Estratégica en Matemáticas

Guillermo Curbera Costello
Coordinator of the Research on
Economical Impact of Mathematics

1. EXECUTIVE SUMMARY

1. 1. MATHEMATICS AND THE ECONOMY

Mathematics constitutes a set of conceptual languages, artificial and symbolic, highly developed for communication between human beings; learning it allows the solution of economic problems vital for the functioning of a society. Therefore it is extremely important for technology and the economy. From the processing of production or consumption data in a computer to the logical reasoning used to justify an economic policy decision, mathematics is present in our everyday reality, enabling the very existence of economic relations. It could be argued that, without mathematical languages and concepts, individuals would be unable to perform the vast majority of economic transactions that take place on a regular basis.

The economic nature of mathematics as a semi-public good is an obstacle to its measurement using the logic of the national accounts, and even more so considering its capacity to derive positive externalities or benefits to society as a whole, which is not necessarily involved in producing it.

However, as a private good, it is characterised by the appropiable benefits that its use generates in economic activities, allowing us to perform a partial approximation of its economic importance. It should not be forgotten that mathematics is present across the whole range of economic sectors, because it affects all sectors and plays a role in the different phases of the production of goods and services, from the design, modelling, simulation and prototyping of products to the optimisation of production and organisational processes and data analysis. Facilitating the generation of benefits or incentives to the exploitation of mathematics in its dimension as a private good necessarily requires the intervention of the public authorities, not least to reduce the learning costs.

The internet revolution has converted mathematics into a fundamental input of production, while services (which account for more than 75% of GDP) have increasingly been incorporating both maths-based physical/technological capital (information and communication technologies, software, electronic devices, etc.), and human mathematical capital.

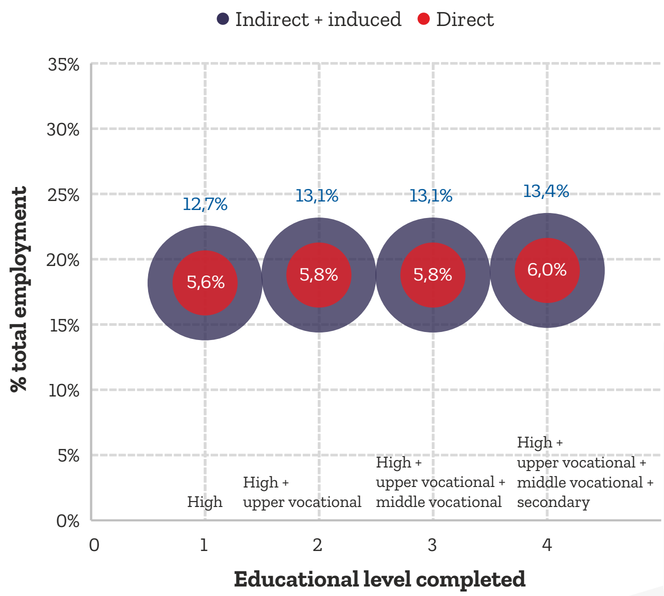
1. 2. THE IMPORTANCE OF MATHEMATICAL RESEARCH AND THE TRANSFER OF MATHEMATICAL TECHNOLOGY IN THE SPANISH ECONOMY

To the extent that mathematics forms part of the stock of both human and physical capital, it can be considered an input (supply). However, there are goods or services whose nature is mathematical (an insurance premium, for example), so that they can also be considered an output (demand). An approach combining both occupations and products reveals that maths-intensive activities generated a million jobs in Spain in 2016, representing 6% of total employment. The addition of indirect and induced employment increases the impact of mathematics in the Spanish labour market to 19.4% of total employment in 2016.

In terms of Gross Value Added (GVA), the impact of maths-intensive activities stood at 10.1% of the total in 2016 (26.9% of the total including carry-over effects). The impact of mathematics is

Estimated direct, indirect and induced impact of maths-intensive activities on employment in Spain (% total employment) by level of education completed, in 2016

Sources: Afi, INE (EPA labour force survey microdata, IOT)



greatest in the IT, finance, telecommunications services, and electricity and gas sectors. These impacts are smaller than those calculated for other European countries where similar studies have been carried out; there, the direct impact in terms of employment ranges between 10% and 11% of the total, whereas in GVA the interval is 13-16% of the total.

The productivity of the economic sectors in which this type of professional worked was 47.2 euros per hour worked in 2016 in Spain, similar to that of comparable countries. Thus, what accounts for the difference in the impact is Spain's economic structure, oriented more toward activities with a lower presence of professions requiring a certain mathematical intensity.

Spain lags notably behind in maths-intensive professions, such as specialists in databases and computer networks, finance, IT and software and multimedia design.

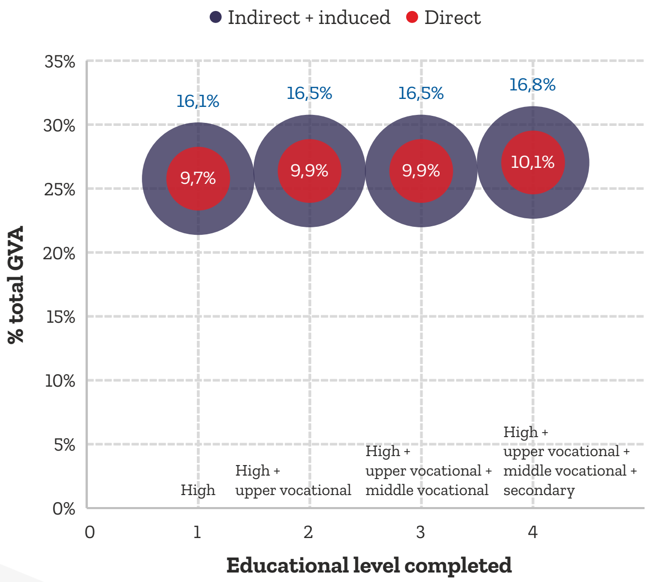
1. 3. MATHEMATICS AS A STRATEGIC VECTOR FOR GROWTH AND ECONOMIC PROGRESS

Mathematics lies at the base of the pyramid of the stock of ideas and knowledge with economic applications. Modern theories of economic growth link long-term per capita income growth to the rate of knowledge accumulation, which induces increases in labour productivity. This is a strategic knowledge for the Spanish economy; its businesses can take advantage of the opportunities arising from the technological revolution associated with robots and artificial intelligence. If Spain increased the proportion of STEM graduates in the total population to the same level as France, labour productivity could increase by 2.2% over current levels.

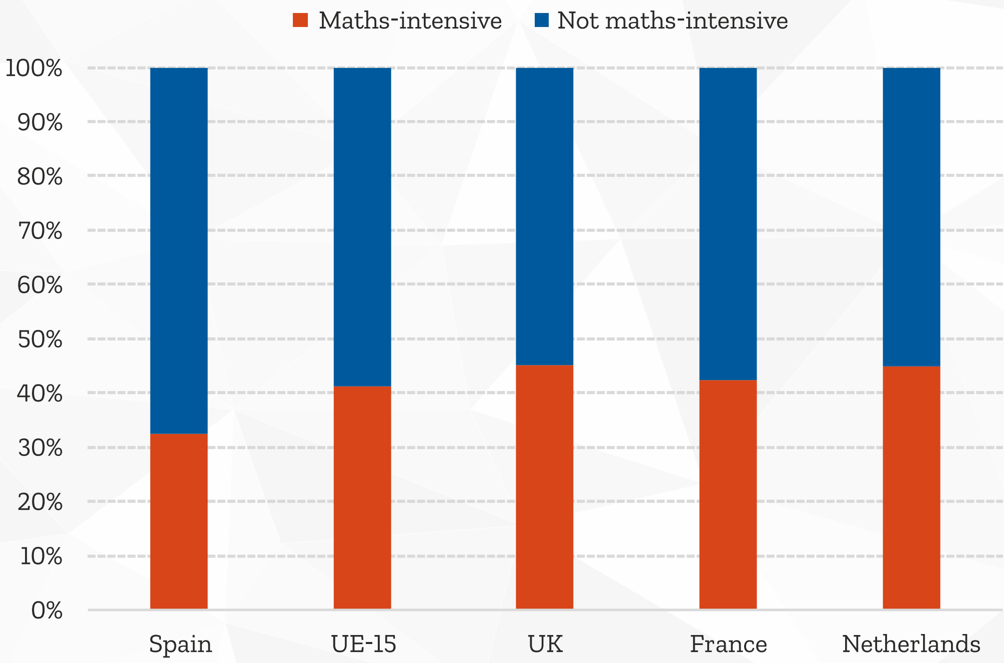
For all this to happen, desirable developments would include (i) mathematics should have a leading role in the educational model, (ii) the linkages between academe and business should be improved, (iii) research and applied mathematics should be enhanced, (iv) and the utility of mathematics in the various phases of the productive process should be made plainly visible to the business community.

Estimated direct, indirect and induced impact of maths-intensive activities on GVA in Spain (% of total) by level of education completed in 2016

Sources: Afi, INE (EPA labour force survey microdata, IOT)



Distribution of employment in maths-intensive occupations of intensity in Spain, the EU-15, United Kingdom, France and the Netherlands (% total employment) in 2015



Sources: Afi, Cedefop

2. INTRODUCTION

Anyone who reflects on the importance of maths will soon notice the difficulties that a society would encounter without it in such basic matters as organising or administering itself. Suffice it to imagine just how costly the functioning of markets would be in the absence of a pricing system, or the coordination of transport systems or the establishment of a communications network based on alphanumeric codes without the seemingly trivial language of numbers.

The continuous accumulation of mathematical knowledge derived from research is a key factor in accounting for both growth and for economic, technological and social development throughout history. Palaeontological research has established that mathematics has served as a tool for the most primitive and ancestral communities, and that there is even evidence[1] of such knowledge prior to the first written records. Later, great civilisations such as Mesopotamia and Egypt applied

1 Henahan, Sean (2002): *Art Prehistory. Science Updates* documents findings of drawings of elementary mathematics in prehistoric communities.

their mathematical knowledge[2] in arithmetic and geometry to address problems including the need to predict harvests and floods, keep records of cattle, construct buildings or plan the size of armies. Such advances served as the basis for continuing the study of geometry in ancient Greece,[3] when other decisive contributions were made in the fields of astronomy and logic, as well as in the application of mathematics to artistic expressions in sculpture and architecture (such as the golden ratio), which were taken up centuries later in the Renaissance era. In short, it can be seen that the universal language of mathematics forms a key part of the cultural heritage of mankind, passing down from generation to generation constituting, as expressed by Raymond Wilder, "a collectively owned good" whose use and application allows the sophistication and modernisation of societies.

The creative intensity of mathematical science has accelerated markedly in the last four centuries, and its application to other disciplines (physics, chemistry, engineering, etc.), as well as to economic activities, has been exponential. In this respect, it is relevant to note that mathematics underlies the emergence and development of the successive technological disruptions that have occurred from the 18th century to our times, which have been called the third industrial revolution (that of the internet), in which the role of mathematics is absolutely central. Indeed, computer science, originated in the middle of the 20th century, algorithms and the analysis of massive databases are the foundations of the most dynamic economic activities of our time. Meanwhile, the horizon marked by advances in robotics and artificial intelligence, by definition intensive in mathematical knowledge, will only continue to boost the role of the science of numbers in the economy.

Accurately defining and estimating the contribution of mathematics to the economy is an arduous task. Due to the nature of its construction, Gross Domestic Product (GDP)[4], the basic measure of economic activity, is of limited use in valuing those goods that do not pass through the market, such as public goods. Thus, the intangible nature of mathematics, which makes it difficult to measure, and, above all, its non-rivalrous character, mean that it is virtually a public good and make it difficult to assess its real utility for society using the logic of national accounting, the general framework for measuring activity and welfare in which GDP constitutes the indicator par excellence.

The aim of this explanation is not to argue that the task is impossible, in spite of the obvious methodological difficulties involved in splitting out the contribution of mathematics to the economic system. Rather it is a sign of caution, to warn the reader ex ante that the work presented starts from less ambitious premises, since the economic value of mathematics can only be measured on the basis of "approximation". In the present work, this "approximation" (there are others, but they fall outside the framework of the national accounting and are based on subjective preference criteria) is that of the appropriable and tangible benefits for the agents that possess the mathematical knowledge and obtain particular benefits from it. This is the approach also followed by the pioneering studies in this area, as is the case of the reports developed in the United Kingdom, the Netherlands and France.

2 Kline, M. (1992): Mathematical Thought From Ancient to Modern Times.

3 Heath, T.L. (1981): A History of Greek Mathematics.

4 GDP is defined as the market value of the final goods and services produced by an economy over a given time period (usually a year). It is also defined by the sum of the added value of the production process. Therefore, GDP is identified with the sum of Gross Value Added (GVA) and indirect taxes on products less subsidies.

The present study is structured as follows. Section 2 characterises mathematics as an economic good and describes the relationship between mathematics and economics. Section 3 approximates the contribution of mathematics to GDP using Gross Value Added (GVA), the share of total employment and tax collection in Spain, and a comparison of this estimate with other countries where similar studies have been carried out. Section 4 analyses the importance of mathematical research and the transfer of mathematical technology for economic growth, through the progress and positive externalities to which it gives rise. Finally, the paper outlines some thoughts that could help to design policies to support the development and applicability of mathematics in Spain, in the light of the evidence provided in the study and the conclusions of several rounds of interviews with individual familiar with the maths ecosystem, ranging from university researchers to professionals in technology companies active both in Spain and outside its borders.

3. MATHEMATICS AND THE ECONOMY

Mathematics constitutes a highly developed set of conceptual languages that are artificial, universal and symbolic. It enables communication between human beings, and learning it allows us to solve economic problems critical to the functioning of a society. However, it establishes a series of rules that differentiates it from everyday language. In particular, mathematics uses a vocabulary of signs that represent numbers, figures or things, together with the relationships between them. It is used for the construction of formulas, equations and mathematical expressions that, with the assistance of connectors (conjunctions, disjunctions, etc.) and logical quantifiers (there is, any, etc.) allow the construction of what in an ordinary language would be sentences. These expressions acquire meaning through their inclusion in theorems and proofs. These represent statements and reasoning, the equivalent of texts or essays.

Without mathematical language, the vast majority of economic transactions could not occur.

Mathematical language is present[5] throughout the economy. From the processing of production and consumption data in a computer to the logical reasoning used to justify economic policy decisions, mathematics is present in everyday life, enabling the very existence of economic relations. It could be argued that, without mathematical language, individuals would be unable to perform the vast majority of economic transactions that take place on a regular basis. In other words, mathematics is an instrument that drastically reduces transaction costs, making possible the interaction between the supply of and demand for goods and services. Hence the interest in understanding the economic nature of mathematics and how it affects economic activities.

Mathematics forms the base of the pyramid of ideas and knowledge applied to economic activities, and requires public intervention if it is to be transmitted to the population in optimal proportions.

3. 1. THE ECONOMIC NATURE OF MATHEMATICS

3. 1. 1. Mathematics: a public club good.

Like other collective productions, such as language[6], the research and intensive use of mathematics has some characteristics of a public good. Once a theorem has been stated or demonstrated, e.g. the fundamental theorem of calculus, its use does not exhaust the possibilities of use by the rest of the human race, irrespective of the time or place in which this was done. This is what is known as "non-rivalrous".

However, unlike other pure public goods, such as air or national defence, mathematics has a certain cost of access, i.e. learning, which restricts entry or excludes those who do not master or know its language. The smaller individuals' predisposition to learn or their capacity to access it, the higher these costs will be. Thus, in line with the principle of exclusion, based on the costs of access, this puts them in the category of a public club good. However, unlike the majority of such public club goods, it does not have congestion economies, but adoption or network economies (the more users it has, the more value it has). That is to say, it never becomes a "rivalrous good" as such, rather quite the opposite.

Its nature as a public club good has diverse implications. Firstly, the market does not have the ability to produce the optimal amount of knowledge transmission or research, as appropriability is lacking and exclusion is

5 An in-depth description can be found in the lessons of Concepción Gonzalez: "Matemáticas como recurso para economía".

6 Fundación Telefónica has conducted several studies analysing different dimensions of the economic value of the Spanish language, such as "The economic value of Spanish", Delgado, Alonso and Jiménez (2012) and "The accounts of Spanish", Girón and Cañada (2009).

Diagram 1. Characteristics of private and public goods

	Excludable	Non-excludable
Rivalrous	**Private goods** Clothing Food	**Common-pool goods** Public parks International waters **Congestible goods** Toll-less highways Public education
Non-rivalrous	**Club goods** Paid television Sports clubs	**Pure public goods** Defence Research

Source: Afi

only partial. This is one of the reasons why the transmission of mathematical knowledge requires public intervention, to ensure that it is provided in optimal quantities (basic education, research, etc.), beyond what the market can allocate.

Mathematics lies at the base of the pyramid of the stock of ideas and knowledge with economic applications. Modern theories of economic growth link long-term growth in per capita income to the rate at which that knowledge is accumulated (as will be seen below, in section 4). Moreover, the economic effect is proportional to the population that can access the use of this resource.

Not being an appropriable resource, there is no market value that reflects its true utility to society. This "market failure" is determined by its status as a quasi-public good and by the fact that national accounting systems can only measure private goods, i.e. those that have been assigned a market price. But another "market failure" that hinders the measurement of its contribution to the economy is that mathematical knowledge generates positive externalities (indirect benefits in other markets that are not reflected in the prices of transactions). For example, the teaching of mathematics allows people to make better decisions (resource management, for example), a utility that is not found in the cost of the transfer of knowledge (the salary received by a maths teacher).

Thus, mathematics as an economic good presents similarities with ordinary language, sharing its four basic characteristics, as shown in the following diagram.

The transmission of mathematical knowledge generates large positive externalities that are not usually recognised in the statistics.

Diagram 2. Characteristics of mathematics as a public good

Source: Afi

3. 1. 2. Mathematics as a private good

Another approach to the analysis of the economic nature of mathematics considers its nature as a private good, taking into account the appropriable benefits of the rewards of mathematical research for agents who possess the knowledge. Mathematics, from this perspective, can be understood as an asset carried by individuals, which requires an initial investment in learning (the access costs mentioned in the previous subheading) and generates a series of flows of tangible benefits for the individuals who acquire it, throughout their lives.

Thus, individuals are faced with the disjunction of "investing" effort in mathematical learning, depending on the balance between the flows of benefits and costs. When these benefits are easily predictable and higher than the costs, individuals will invest in their mathematical education.

The smaller the learning costs and the higher the benefits remunerating the knowledge, the greater will be the incentives to extend the stock of mathematical knowledge among the individuals of a given community.

The costs of learning include both the monetary costs and the value (opportunity cost) of the hours of effort needed to attain the standard required, depending on the discipline in question. With regard to the benefits, these could be expressed as the difference in salary over the working life (as measured by t) of the workers who buy this knowledge, compared to the salary that they could expect without possession of that knowledge.

$$\sum_{t}^{T} \frac{Benefits_t}{(1+r)^t} - Learning\ costs$$

This simple formula leads to a series of significant conclusions. To cite just one, the greater the public support for reducing the costs of learning, the greater the incentive to extend the stock of mathematical knowledge among the individuals of a given community. With regard to the benefits, the higher the wage premium remunerating that knowledge, the greater the incentives for individuals to "participate" by undertaking mathematical learning.

The advantage of this approach to the nature of mathematics as an economic good is that it can be reconciled with the metrics available in the national accounts, inasmuch as the "salaries and benefits" attributable to mathematics are observable in the statistical information prepared by public institutions.

Mathematics is applicable across economic activities and is increasingly present in the services sector, which accounts for about 70% of GDP.

3. 2. WHY IS MATHEMATICS IMPORTANT FOR ECONOMIC ACTIVITIES AND THE ECONOMY?

> *"The economic world is a misty region. The first explorers used unaided vision. Mathematics is the lantern by which what before was dimly visible now looms up in firm, bold outlines. The old phantasmagoria disappear. We see better. We also see further".*
>
> — Irving Fisher, *economist*

Among the so-called social sciences, economics is that which deals with the making of decisions and the interaction between them. From this perspective, mathematics constitutes a fundamental tool box to improve decision-making, as well as a fundamental language that, as noted above, eliminates or reduces certain transaction costs. In fact, mathematical programming is a basic instrument that provides us with rules of logical reasoning that enable us to achieve one of the chief aims of economic science, the "efficient allocation of scarce resources". On the other hand, such basic fields of mathematics as matrix algebra are extremely useful for the presentation and processing of information in multiple areas, from econometrics to accounting, as well as for the achievement of better business management. Probabilistic models, in turn, are a fundamental instrument for dealing with the decisions in contexts of uncertainty and risk, for example, in the financial field.

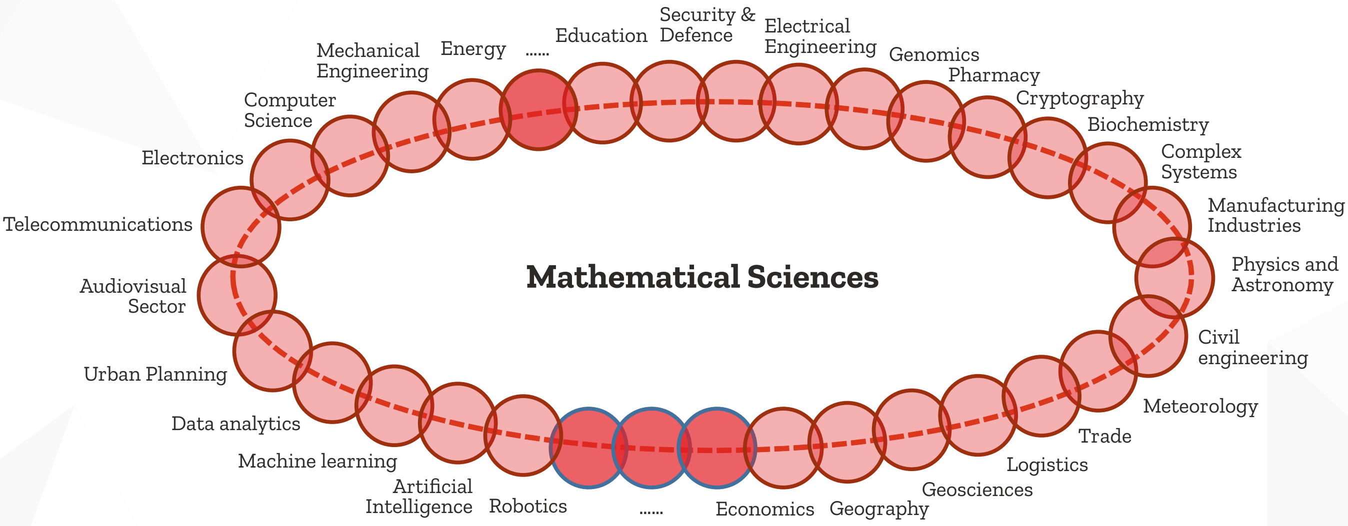
In any case, mathematics, in its different approaches and facets, is applicable across the whole range of economic activities. Until a few decades ago, its role was predominantly in industry and construction, given the maths-intensive nature of the various branches of engineering (industrial, roads and ports, etc.). However, the internet revolution has converted mathematics into a fundamental input of production, while services, which account for close to 70% of the economy's final production in developed economies, have increasingly been incorporating both maths-based physical-technological capital (information and communications technologies, software, electronic devices, etc.), and human mathematical capital, such as that used to analyse data for different business functions, from advertising or marketing strategies, to business strategies (pricing) or the optimisation of resources (human, energy, etc.). In general terms, the participation of mathematics in the economy could be summed up in three key aspects:

1) **DESIGN, MODELLING, SIMULATION AND PROTOTYPING OF PRODUCTS.** Mathematics adds a great deal of precision to the production of goods and services, allowing them to be made more sophisticated. The most frequent illustrative example is the automobile industry, where mathematics has been an essential tool for the improvement of vehicles.

2) **OPTIMISATION OF PRODUCTION PROCESSES AND ORGANISATION.** Mathematics is essential to reduce internal and external transaction costs and improve efficiency (production at the lowest possible cost). An obvious example is the application of mathematics to industrial logistics, for the optimisation of the distribution network (warehouse, distribution, route, schedule planning, travel, etc.).

3) **DATA ANALYSIS.** The tools provided by mathematics are essential to make economic sense of the information available and to take advantage of it (e.g. using Big Data analytical techniques). In the aerospace sector, for example, the creation of analysis platform prototypes for large volumes of unstructured data enables the efficiency of flights (fuel consumption, emissions, flight times, delays, etc.) to be improved.

Diagram 3. Some links between the mathematical sciences and other academic disciplines and economic sectors



Source: The Era of Mathematics

4. THE IMPORTANCE OF MATHEMATICAL RESEARCH AND THE TRANSFER OF MATHEMATICAL TECHNOLOGY IN THE SPANISH ECONOMY

As noted above, the fact that mathematical research and technology transfer (hereinafter, MRT) does not have a market value makes it difficult to measure its importance in the Spanish economy. The methodological approach used in this work has been widely used in similar projects[7] and consists in the consideration of the appropriable benefits for those agents who possess the mathematical knowledge and receive its rewards in their corresponding activities.

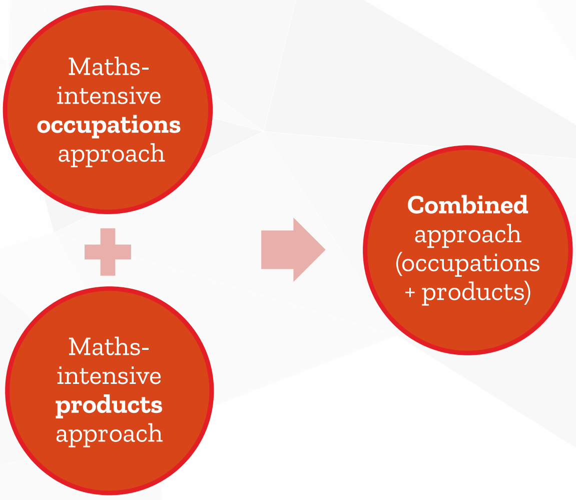
7 The development of this study has referred to reports prepared by Deloitte in the cases of the UK (2012) and the Netherlands (2014), and by the CMI in the case of France (2015).

4. 1. THE QUANTIFICATION OF MATHEMATICS AS AN ECONOMIC GOOD

To the extent that the MRT is part of the stock of human capital and is also incorporated in the stock of physical capital, it can be considered as an input (supply). However, there are goods or services whose nature is mathematical (an insurance premium, for example), so that they can also be considered an output (demand). Therefore, its contribution may be estimated using both approaches, with the key being to discern the maths intensity, of both the inputs and the outputs.

The analysis developed in this work is based (see diagram 4) on the occupations approach (supply). However, it also explores the products approach (demand), as well as the possibility of taking advantage of their complementarity (combined approach).

Diagram **4. Proposed methodology to estimate the importance of the MRT in the Spanish economy**



Source: Afi

4. 1. 1. Quantitative estimate using the maths-intensive occupations approach

Based on the occupations approach (see appendix), in Spain there were between 2.4 and 3.8 million full-time equivalent (hereinafter, fte) employed maths-intensive workers in 2016, according to the level of completed studies (see Table 1). These quantities represent between 14.1% and 22.1% of the total employment in the Spanish economy in that year.

In both cases, this is an estimate of the maximum, as it considers that these workers spend all their working time performing maths-intensive tasks. However, this may not be the case, because there are certain tasks, also relevant in the exercise of any profession, which do not require advanced mathematical knowledge from workers or do not rely on tools with high mathematical content (machinery). An example of this is face-to-face commercial and institutional relationships, attendance at team coordination meetings, etc. Nevertheless, in many

Table 1. **Total maths-intensive employment and FTE according to educational level completed (number and % total) in 2016**

No. Workers	High	High+ upper vocational	High+ upper vocational + middle vocational	High+ upper vocational + middle vocational + secondary
Total				
By educ. level	2.350.916	568.165	1.261	801.724
Cumulative	2.350.916	2.919.080	2.920.341	3.722.065
FTE				
By educ. level	2.450.880	582.801	1.401	805.940
Cumulative	2.450.880	3.033.681	3.035.083	3.841.022

% total	High	High+ upper vocational	High+ upper vocational + middle vocational	High+ upper vocational + middle vocational + secondary
Total				
By educ. level	13,5%	3,3%	0,0%	4,6%
Cumulative	13,5%	16,8%	16,8%	21,4%
FTE				
By educ. level	14,1%	3,3%	0,0%	4,6%
Cumulative	14,1%	17,4%	17,4%	22,1%

Sources: Afi, INE (EPA microdata)

of these tasks the use of time and optimisation of the same are also taken into account.

Therefore, it is desirable to allocate a proportion of time to each of the maths-intensive occupations identified. Statistics on the use of professional time are not available in the microdata from the Labour Force Survey (EPA) published by the National Institute of Statistics, making it necessary to turn to experts[8] in maths transfer (see panel in the appendix section) and in the tasks performed by this type of professionals in their daily work.

8 The figures shown below were calculated using the arithmetic averages of the proportion of maths-intensive time of each of the occupations estimated by the experts consulted. To avoid excessive differences of views, the answers considered outliers were removed, i.e. those that exceed the arithmetic average by two standard deviations, both at the lower and upper end. In general, the technical occupations were assigned a higher proportion of time devoted to mathematics than those where the administrative and/or management component is more significant.

According to the occupations approach, mathematics would be responsible for more than 630,000 jobs (3.6% of total employment) and for a contribution of some €62 billion to GVA (6.1% of the total of the Spanish economy in 2016).

Taking into account the above, the results indicate that maths-intensive activities would contribute to the creation of close to 630,000 FTE jobs directly considering the whole group, irrespective of their educational level, equivalent to 3.6% of total employment in the Spanish economy (see Table 2). If the impact is confined to employed workers who have completed a high educational level, then maths-intensive activities would contribute to the generation of more than half a million jobs.

Table 2. **Estimated direct impact of maths-intensive activities on employment in Spain according to level of education completed by the worker (full-time equivalents and % total employment) in 2016**

	High	High + upper vocational	High + upper vocational + middle vocational	High + upper vocational + middle vocational + secondary
No. Workers	507.641	581.257	581.597	629.239
% total	*2,9%*	*3,3%*	*3,3%*	*3,6%*

Sources: Afi, INE (EPA microdata)

The information on productivity per hour worked in each of the 63 economic sectors, provided by the input-output framework (see methodological appendix), together with the estimate of the number of FTE employed, enables the Gross Value Added (GVA) generated by maths-intensive activities to be calculated.

The GVA could even be underestimated if the average productivity of the sector is taken, rather than a higher figure that could be assumed for maths-intensive occupations. Information on the salaries of these workers in each economic sector could be used to quantify this difference, if it existed. However, the absence of statistics on the wage variable in the detailed EPA microdata on the above-mentioned occupations and sectors (+++ three digits in both cases), hinders the confirmation of this fact. Therefore, the only option is to assign each maths-intensive worker the average productivity of the economic sector in question.

Hence, taking account of the entire group of maths professionals, maths-intensive activities would generate about €62 billion in GVA directly, equivalent to 6.1% of the total GVA of the Spanish economy in 2016 (see Table 3). If only those who have completed university studies are considered, the direct impact on GVA would exceed €50 billion (5% of the total).

The productivity of the economic sectors in which this type of professional works was €47.20 per hour worked in 2016, while the overall average is €31.40.

It should be noted that the impact in terms of GVA is higher than that for employment. This is because this type of professionals predominates in the sectors with the highest labour productivity. The average productivity of the economic sectors in which this type of professionals work was €47.20 euros per hour worked in 2016, while the average for all sectors was €31.40 euros per hour worked. Therefore, the economic contribution of the maths-intensive activities is higher than the proportion of workers involved. This helps to improve labour productivity, another weakness of the Spanish labour market, and could enhance the longed-for competitiveness of the Spanish economy.

Table 3. **Estimated direct impact of maths-intensive activities on GVA in Spain by the level of education completed by the worker (€ millions and % total) in 2016**

	High	High + upper vocational	High + upper vocational + middle vocational	High + upper vocational + middle vocational + secondary
Mill. Eur.	50.324	57.406	57.466	61.747
% total	5,0%	5,7%	5,7%	6,1%

Sources: Afi, INE (EPA microdata)

4. 1. 2. Quantitative estimate using the combined approach of occupations and maths-intensive products

As discussed above, the occupations approach considers the role of maths professionals in the economy and their economic impact in terms of added value. However, there are times when their contribution in terms of employment can be important and yet not fully capture their potential value generation. In contrast, the products approach proposed in the methodology may underestimate their contribution in those branches of economic activity where mathematics has a smaller presence.

Thus, the occupations and products approaches have a significant degree of complementarity. However, both procedures cannot simply be aggregated, as there would be a risk of duplication in the aggregates. In order to integrate the two methods, the following points must be taken into account:

(i) For those sectors highly specialised in mathematical products, the products approach will be used, and not the occupations approach.

(ii) In those with less technological specialisation, the occupations approach alone will be used to estimate the impact of mathematics.

The combined approach calculates an impact of more than a million jobs (6% of total employment) and €103 billion of GVA (10.1% of the total).

The combined maths-intensive occupations and products approach shows that the direct impact on FTE employment would rise to 1 million jobs, irrespective of the level of studies completed by the workers. This would represent about 6% of total employment in the Spanish economy in 2016 (see Table 4).

With regard to the direct impact on GVA, the estimate points to an amount of approximately €103 billion, if it all maths professionals are considered, which would represent 10.1% of Spanish GVA in 2016 (see Table 5). Considering only those who have completed higher education, the direct impact would be in excess of €98 billion (9.7% of total GVA).

Table 4. **Estimated direct impact of maths-intensive activities on employment in Spain by level of education completed by the worker (number of FTE and % total employment) in 2016**

	High	High + upper vocational	High + upper vocational + middle vocational	High + upper vocational + middle vocational + secondary
No. Workers	977.425	1.015.549	1.015.552	1.044.965
% total	5,6%	5,8%	5,8%	6,0%

Sources: Afi, INE (EPA microdata)

Table 5. **Estimated direct impact of maths-intensive activities on GVA in Spain according to level of education completed by the worker (€ million and % total GVA) in 2016**

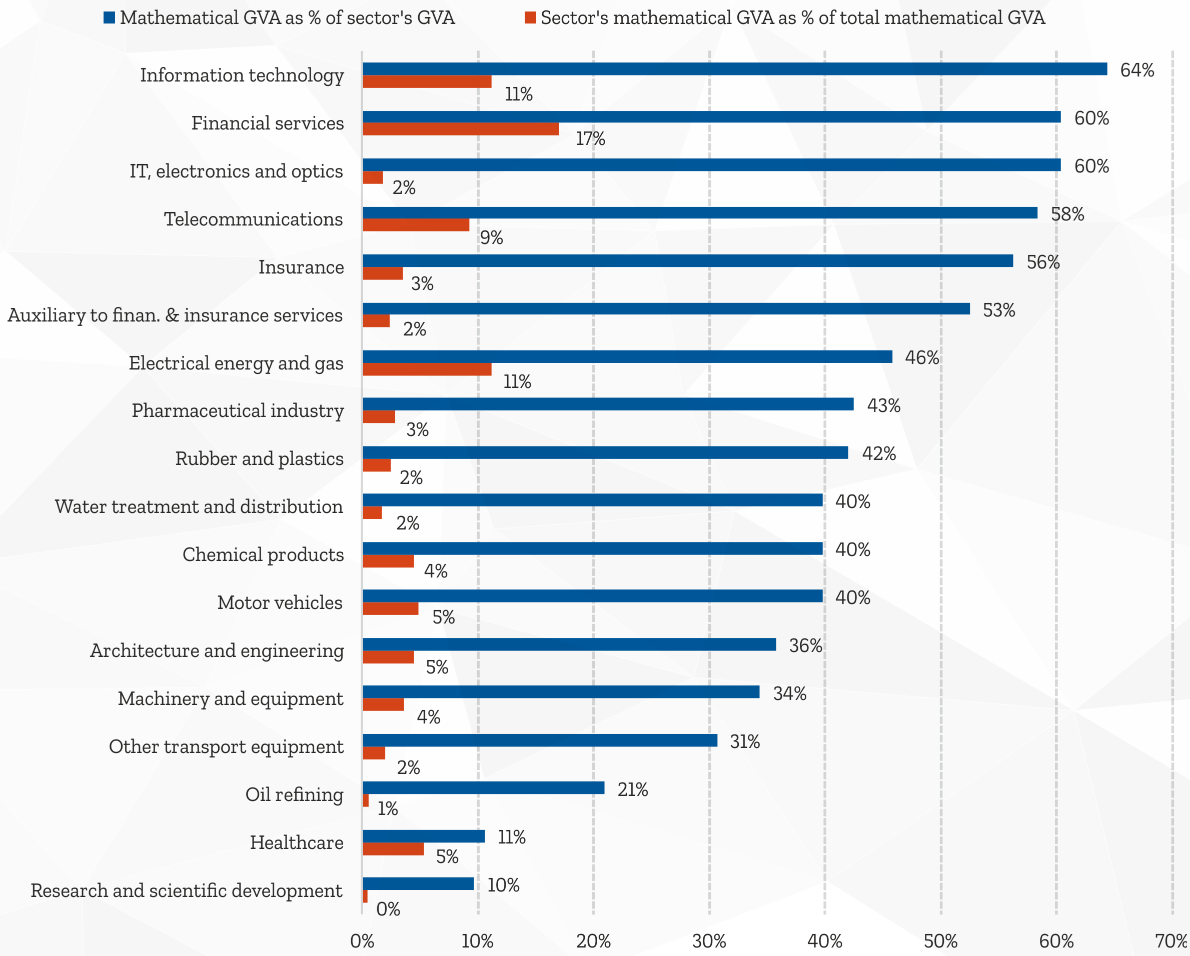
	High	High + upper vocational	High + upper vocational + middle vocational	High + upper vocational + middle vocational + secondary
Mill. Eur.	98.043	100.886	100.886	102.812
% total	9,7%	9,9%	9,9%	10,1%

Sources: Afi, INE (EPA microdata)

Sector by sector analysis reveals that it is computer science, followed by financial activities, telecommunications services, and electricity and gas, where mathematical research and the use of the mathematical tools have the greatest direct impact. Not only is a significant proportion of their GVA generated by mathematics, but they also account for the bulk of its impact on the Spanish economy (see Chart 1).

IT, followed by financial activities and telecommunications services, are the most maths-intensive sectors, and those which account for the bulk of the impact on the Spanish economy.

Chart 1. Economic sectors by size of direct impact of mathematics in 2016



Sources: Afi, INE (microdatos EPA)

Maths-intensive activities are estimated to have contributed about €107 billion in taxes and Social Security contributions, equivalent to 25.8% of the total collected by government authorities in 2016.

Lastly, the average effective rates of the major taxes[9] and the average contribution to Social Security can be used to estimate the revenue accruing to the state from maths-intensive activities.

Using this methodology and the estimates of the direct impact on GVA and employment from the combined approach, maths-intensive activities would contribute about €107 billion in taxes and Social Security contributions, equivalent to 25.8% of the total collected by the government authorities in 2016, considering all workers (see Table 6). If the analysis is restricted to those who have completed higher education, the tax revenues are estimated at €102 billion (24.7% of the total).

9 Personal Income Tax (IRPF), Value Added Tax (VAT) and Corporation Tax (IS).

Table 6. **Estimated direct impact of maths-intensive activities on tax collection in Spain by the level of education completed by the worker (€ million, % total) in 2016**

	High	High + upper vocational	High + upper vocational + middle vocational	High + upper vocational + middle vocational + secondary
Mill. Eur.	102.269	105.246	105.246	107.076
% total	*24,7%*	*25,4%*	*25,4%*	*25,8%*

Sources: Afi, INE

4. 1. 3. Indirect and induced impacts: carry-over effects on other economic sectors

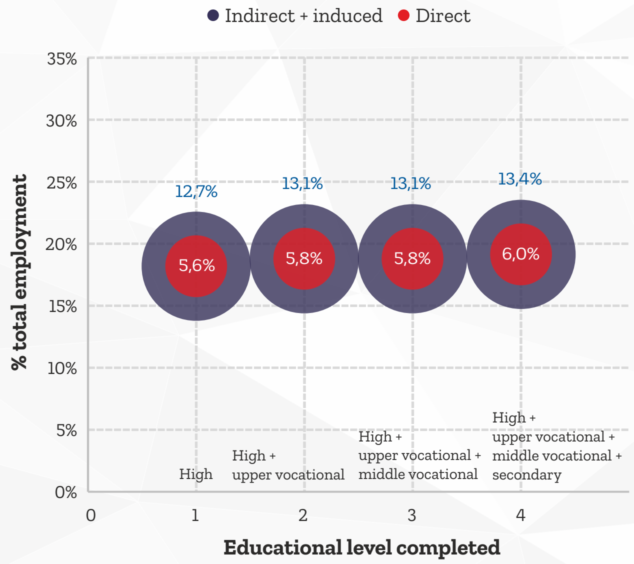
Maths-intensive activities have other impacts beyond those generated directly on workers, the economic sectors in which they work and the technologies that they use in their jobs. These other impacts have to do with (i) those derived from the fact that a sector forms part of a value chain (e.g. the relationship with suppliers of goods and services), i.e. the indirect effects; and (ii) those generated by the use of the income derived from maths-intensive activities (spending and investment of the income from work performed on goods and services by maths-intensive professionals), which would be the induced effects.

Including the indirect and induced effects, maths-intensive activities accounted for around 19% of total employment and 27% of GVA in 2016.

To estimate the indirect and induced impact, multipliers derived from the 2010 input-output table of the Spanish economy, provided by the INE, were used. If these knock-on effects are applied in the estimation of the direct impact deduced using the combined occupations and products approach, maths-intensive activities are seen to contribute to the creation of more than three million jobs, irrespective of the workers' level of education completed (see Chart 2).

***Chart 2.* Estimated direct, indirect and induced impact of maths-intensive activities on employment in Spain (% total employment) by level of education completed, in 2016**

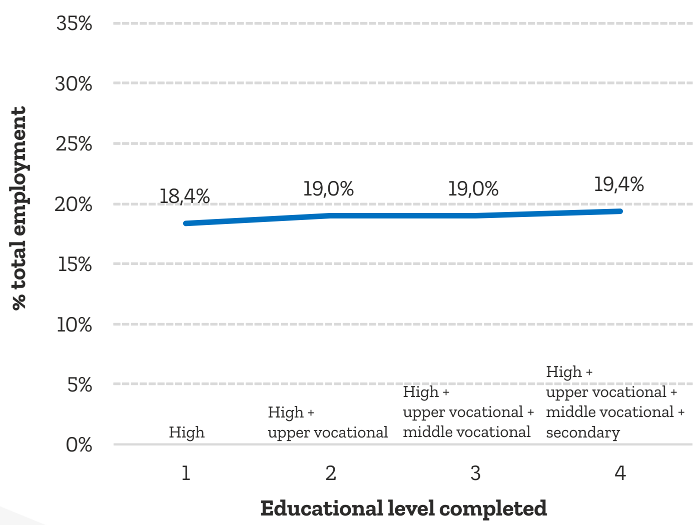
Sources: Afi, INE (EPA labour force survey microdata, IOT)



In relative terms, maths-intensive activities would account for about 19% of the total employment in the Spanish economy in 2016 (see Chart 3). The portion attributable to the indirect and induced impact, therefore, would be about 2.3 million more FTE jobs, distributed among all economic sectors, representing 13.4% of total employment in 2016. With regard to GVA, adding in the indirect and induced impacts of maths-intensive activities, the overall impact would amount to €273 billion euros, 26.9% of the total GVA generated by the Spanish economy in 2016 (see Charts 4 and 5).

***Chart 3.* Estimated total impact of maths-intensive activities on employment in Spain (% total employment) according to level of education completed in 2016**

Sources: Afi, INE (EPA labour force survey microdata, IOT)



In this case, the indirect and induced impacts would generate some €170 billion euros in other economic activities, 16.8% of the total GVA of the Spanish economy in 2016. The magnitude of these impacts highlights the important knock-on effects of maths-intensive activities on the Spanish economy. A breakdown of the forward and backward knock-on effects on the different economic sectors affected by maths-intensive activities can be classified according to the types of these "pull effects" or linkages.

Chart 4. Estimated direct, indirect and induced impact of maths-intensive activities on GVA in Spain (% of total) by level of education completed in 2016

Sources: Afi, INE (EPA labour force survey microdata, IOT)

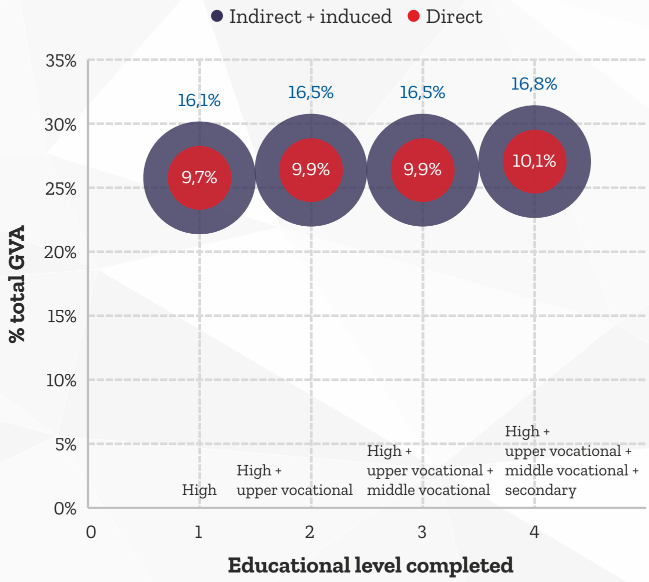
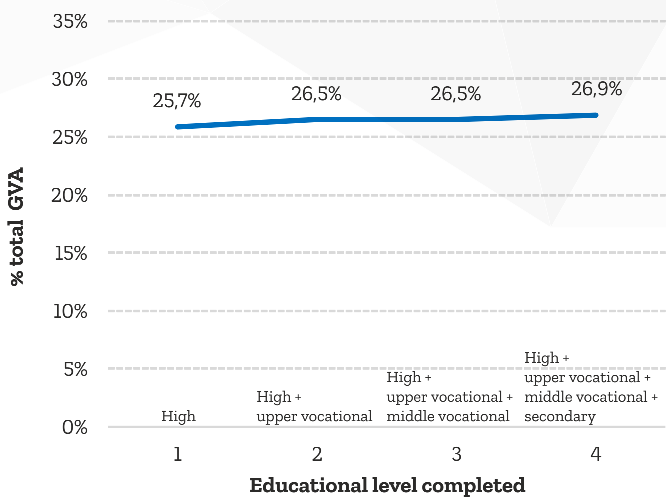


Chart 5. Estimated total impact of maths-intensive activities on GVA in Spain (% of total) according to level of education completed in 2016

Sources: Afi, INE (EPA labour force survey microdata, IOT)



The most maths-intensive activities constitute key sectors, both in terms of their knock-on capability and their support for the rest of the economy.

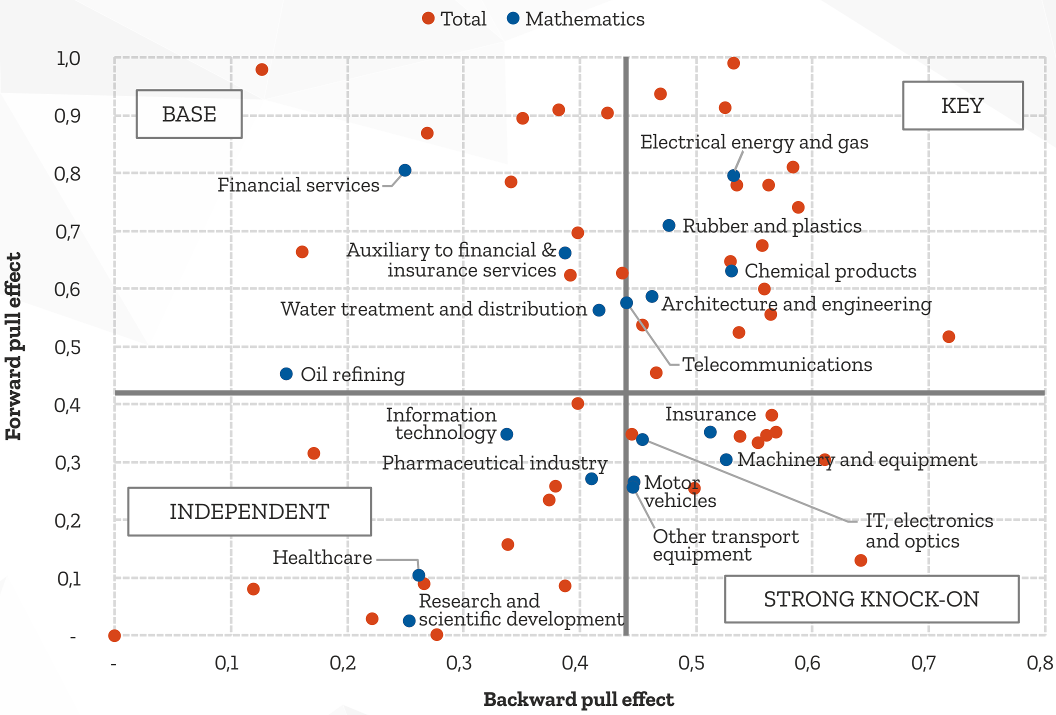
In Chart 6, the activities with strong forward and backward linkages, considered "key" sectors, are concentrated in quadrant II (top right). They are so called because they exert both a knock-on effect on other activities and support the growth of other sectors; hence they are strategic in nature. In this quadrant, we find the electricity and gas sector, rubber and plastics industries, chemical industry, architectural and engineering services and telecommunications services.

All these sectors are the same ones that present the highest overall impacts of intensity in mathematics. However, the rest of the economic sectors with above-average impact of mathematics, such as financial services, information technology or R&D&i activities, are distributed among the four quadrants, evidencing their cross-cutting nature.

4. 2. THE ECONOMIC IMPORTANCE OF MATHEMATICS IN SPAIN IN COMPARATIVE PERSPECTIVE

Another way to assess the importance of mathematics in the Spanish economy is to compare it with the situation in other developed European economies. This analysis also shows the progress made by maths-intensive activities in Spain. Judging by the views shared by a good part of the experts consulted, this has been extensive and praiseworthy. However, it also shows what remains to be done, since the impact of maths-intensive activities as estimated above is lower than that calculated for other European countries where similar studies have been carried out.

Chart 6. Classification of the maths-intensive sectors according to their backward and forward pull effects in 2016



Sources: Afi, INE (EPA labour force survey microdata, IOT)

The importance of maths-intensive occupations is lower in Spain than in the United Kingdom, France and the Netherlands.

In terms of direct employment, it should be noted that the estimated impact in Spain is 6% of total employment according to the combined approach, while figures for the United Kingdom, France and the Netherlands range between 10% and 11% (see Table 7).

The same is true in terms of GVA, as the estimate for Spain is 10.1% of the total, while in other European countries the direct impact is between 13% and 16% of total GVA (see Table 8).

Table 7. **Estimated impact of maths-intensive activities on employment in the United Kingdom, France, the Netherlands and Spain (millions of FTE and % total employment)**

Employment (thous.)	Direct	Indirect	Induced	Total
UK	2,8	2,9	4,1	**9,8**
France	2,4	n.d.	n.d.	n.d.
Netherlands	0,9	0,5	0,8	**2,3**
Spain	1,0	2,3		**3,3**

Employment (% total)	Direct	Indirect	Induced	Total
UK	9,8%	10,2%	14,4%	**34,4%**
France	9,0%	n.d.	n.d.	n.d.
Netherlands	10,7%	6,2%	9,1%	**26,0%**
Spain	6,0%	13,4%		**19,4%**

Sources: Afi, INE (EPA microdata)

Table 8. **Estimated impact of maths-intensive activities on GVA in United Kingdom, France, the Netherlands and Spain (€ billions, % total)**

GVA (billions)	Direct	Indirect	Induced	Total
UK	208	155	192	**555**
France	285	n.d.	n.d.	n.d.
Netherlands	71	37	51	**159**
Spain	103	170		**273**

GVA (% total)	Direct	Indirect	Induced	Total
UK	16,0%	12,0%	15,0%	**43,0%**
France	15,0%	n.d.	n.d.	n.d.
Netherlands	13,2%	6,9%	9,5%	**29,6%**
Spain	10,1%	16,8%		**26,9%**

Sources: Afi, Deloitte, CMI

Replicating the exercise carried out in these other European countries, using the occupations approach (and not the combined approach) and assigning the average proportion of hours assigned to each occupation (instead of that estimated by the experts consulted), the direct impact on employment and GVA (6.4% and 10.7% of the total, respectively) would also be lower than that observed in the United Kingdom, France and the Netherlands (see Table 9).

It is not at all surprising that this impact is lower in Spain that the quantified by the studies consulted in other European countries. This is due to the different structure of the Spanish economy (there are fewer maths-intensive employed workers, representing a lower proportion of total employment than in these other countries), while the economy as a whole is less competitive (lower labour productivity in relative terms).

Table 9. **Estimated direct impact of maths-intensive activities in Spain, assigning the same proportion of time as the studies of United Kingdom, France and the Netherlands**

Impact on employment (FTE employment, % total) in 2016

	High	High + upper vocational	High + upper vocational + middle vocational	High + upper vocational + middle vocational + secondary
No. Workers	850.635	1.037.990	1.039.170	1.119.966
% total	*4,9%*	*6,0%*	*6,0%*	*6,4%*

Impact on GVA (€ billion of GVA, % total) in 2016

	High	High + upper vocational	High + upper vocational + middle vocational	High + upper vocational + middle vocational + secondary
Mill. Eur.	82.142	100.642	100.895	108.859
% total	*8,1%*	*9,9%*	*9,9%*	*10,7%*

Sources: Afi, INE (EPA microdata), Deloitte, CMI

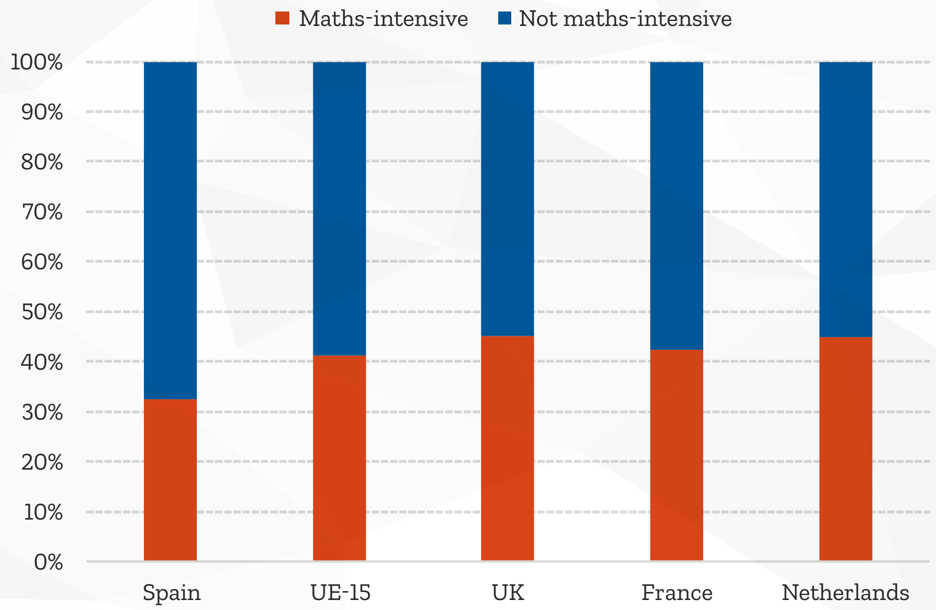
Spain is notably behind in maths-intensive occupations, such as specialists in databases and computer networks, finance, information technologies, and software and multimedia design.

According to Eurostat, the importance of maths-intensive occupations is lower in Spain than in the United Kingdom, France and the Netherlands. Aggregating in terms of their maths-intensity[10], it can be seen that maths-intensive occupations represent around 30% of the total in Spain (see Chart 7), while in the EU-15 this same percentage is 40%, being even higher in the case of the UK, the country with the highest estimated impact among those analysed.

10 These data are less detailed than the EPA microdata used in the previous estimates; hence, the resulting figure may be overestimated.

Within these maths-intensive occupations, Spain is notably behind the UK, France and the Netherlands in the following professions:

***Chart 7.* Distribution of employment by maths-intensive occupations in Spain and the EU-15, United Kingdom, France and the Netherlands (% total employment) in 2015**



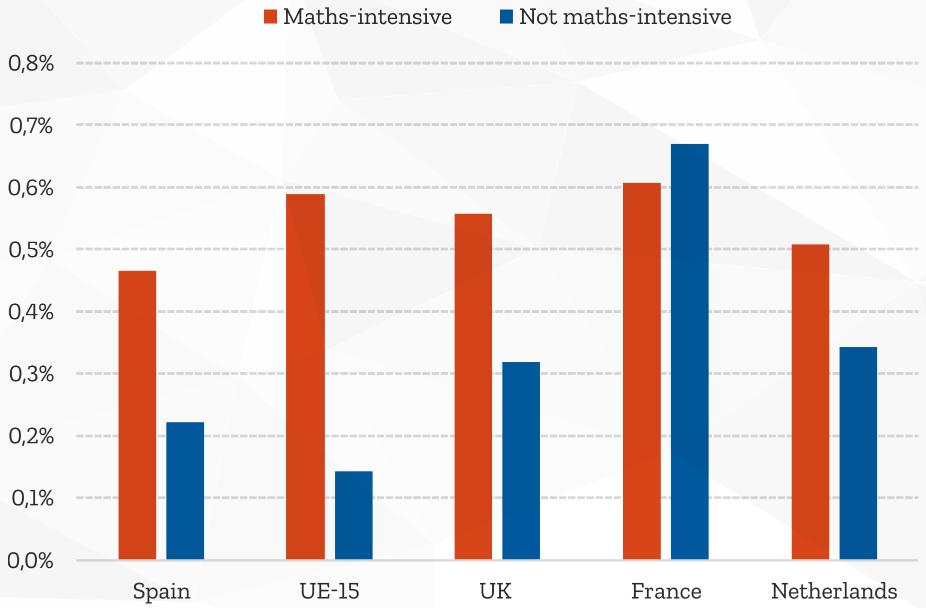
Sources: Afi, Cedefop

- Specialists in organisation of public administrations and companies and in marketing; these include occupations such as finance specialist (code 261 of the Spanish National Occupations Classification of 2011, hereinafter CNO-11), specialists in organisation and administration (code 262) and technical and medical sales staff, except ICT (code 264). This aspect is unsurprising, since the European financial institutions, as well as the centres of economic power, are distributed between the City of London and the cities located in the centre of the European continent (Paris, Luxembourg, Brussels, etc.).
- Information technology (IT) workers, such as software and multimedia analysts and designers (code 271) and specialists in databases and networks (code 272). Even though ICT technicians (programmers, audiovisual and telecommunications technicians, etc.) have a greater relative importance in Spain than in these other European countries, what is certain is that ICT engineers have a smaller presence in the Spanish economy.

The growth forecast for maths-intensive occupations in the next decade is lower for Spain than the leading European countries.

Moreover, according to Cedefop, a European Commission agency, forecasts for the growth in maths-intensive occupations for the 2015-2025 decade are lower for Spain (estimated to grow at an average annual rate of 0.47%) than for these other European countries (average annual rate of 0.59% for the EU-15, for example), which would widen the already existing gap (see Chart 8). This is despite the fact that the employment growth prospects of

Chart 8. **Growth in employment by maths-intensive occupations in Spain and the EU-15, the UK, France and the Netherlands (average annual growth). Cedefop 2015-25 forecasts**



Sources: Afi, Cedefop

The difference in the impact of maths-intensive activities is due rather to differences in economic structure than to lower labour productivity.

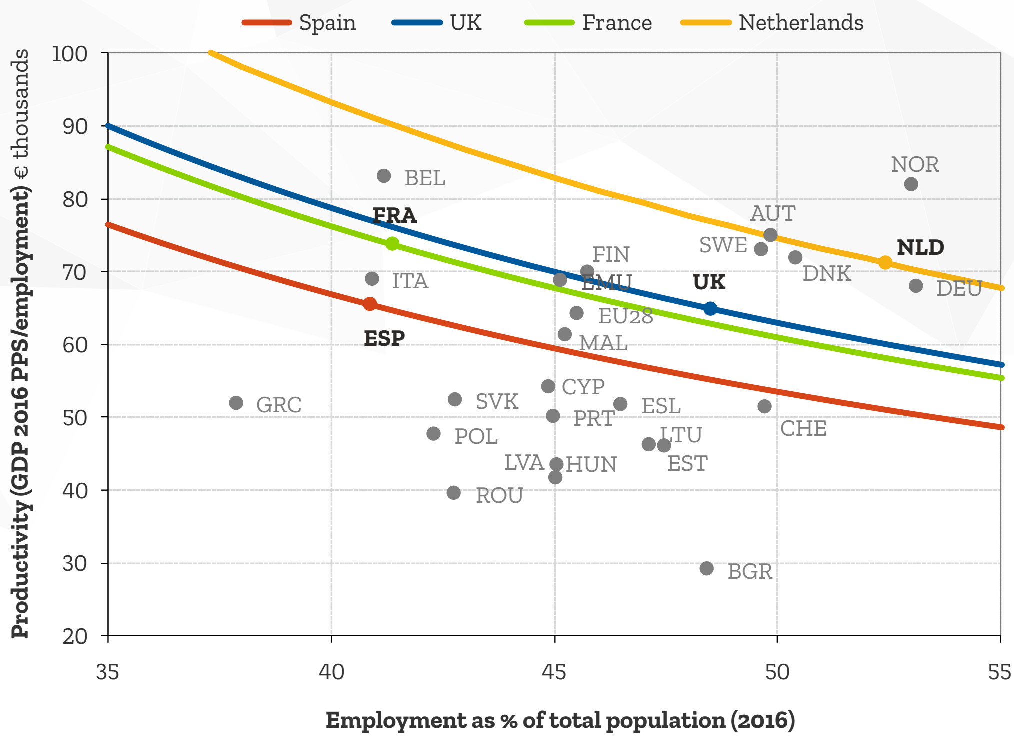
these professions for the next decade are much more favourable than for other occupations. In particular, their growth rate (of 0.47% as noted above) is more than double that of the rest of occupations (annual average of 0.22%) for the 2015-2025 period.

The productivity generated by each employed worker in Spain is substantially lower than that of the UK, France and the Netherlands. Indeed, the underutilisation of human resources (lower rate of employment) and lower labour productivity (GDP per employee) are sometimes cited as two elements limiting per capita GDP growth[11] (see Chart 9). However, the gap between the impact on employment and GDP from one country to another is practically the same, which suggests that, in one way or another, the productivity of maths professionals in Spain a relative to their national average is very similar to that observed in these other European countries. Therefore, the difference in the impact of maths-intensive activities between Spain and the UK, France and the Netherlands is due rather to the different economic structure than to lower labour productivity.

11 Per capita income = labour productivity x percentage of population in employment:

$$\frac{\text{GDP}}{\textit{Population}} = \frac{\text{GDP}}{\textit{Employment}} \times \frac{\textit{Employment}}{\textit{Population}}$$

Chart 9. **Breakdown of GDP per capita in major European countries (€ thousands* and % of total population, iso-income curves per inhabitant**) in 2016**



* At Purchasing Power Standard (PPS).
** Iso-income per inhabitant curves are the common ground of all the combinations of productivity and employment rate that produce the same per capita income, indicating that there are different ways of reaching a certain level of well-being by combining these factors, which are its two pillars.

Sources: Afi, INE, Eurostat

5. MATHEMATICS AS A STRATEGIC VECTOR FOR GROWTH AND ECONOMIC PROGRESS

Mathematical knowledge constitutes one of the bases for achieving improvements for the combination of productive factors, thus increasing the productive capacity of the economy. In other words, it is an essential tool for boosting the productivity of labour, the key determinant of economic growth over the long term[12] (GDP per capita), when all the factors of production are organised for the production of goods and services. Moreover, as it is not a rivalrous good, the economic benefits of its use can be proportional to the population, thus promoting very rapid growth, such as has been observed in China over the past three decades.

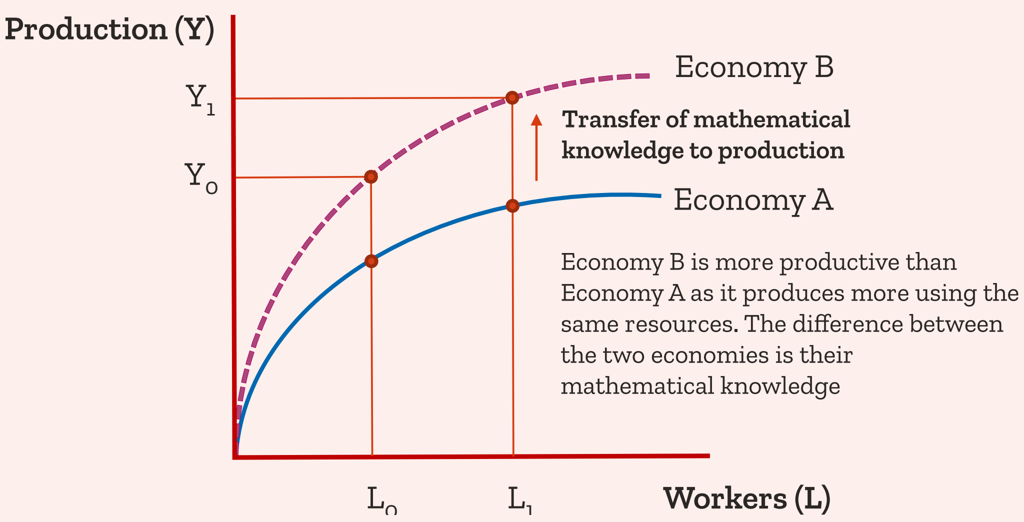
12 Paul Krugman said that while labour productivity is important in the short term, "in the long term it is almost everything".

Mathematics and economic growth

Using a simple ratio of the number of workers in an economy to the total output of goods and services, it can intuitively be seen why the stock of mathematical knowledge and its transfer to productive activities is a critical determinant of the capacity for long-term growth.

Chart 10 compares two imaginary identical economies (both with the same number of workers), identical except with respect to their mathematical knowledge (economy B incorporates more maths into the productive process than economy A). The workers in economy B are able to produce a greater quantity of goods and services in the same amount of time as workers in economy A, simply because their "production techniques" are more sophisticated. A concrete example would be to assume that the economy B workers know and can use algorithms, while those of the economy do not. The algorithms, in short, would solve allocation problems very efficiently for workers in economy B, saving hours of work otherwise required to achieve the same level of output and increasing the population's GDP per capita.

Chart 10. **Difference in productive capacity between two economies due to mathematical knowledge**



Source: Afi

Mathematics is located at the base of the increase in labour productivity, the major determinant of economic growth over the long term.

The transmission channels of mathematical research to productivity are varied. On the one hand, they have a decisive impact on a series of enabling infrastructures, from the construction of housing and physical infrastructure to basic goods such as education or health, utilities (water supply, energy, etc.), security, transport and information and communication technologies themselves. Such infrastructures are essential to increase the population, its life expectancy and its employability. On the other hand, they are also essential for the development of technical skills. As noted at the beginning of the report, an investment is also required to take advantage of mathematics. It is estimated that the improvement in mathematics education from an early age generates a future return in terms of better jobs and higher wages of between 7% and 10%.

In addition, it is fundamental for the generation of business ecosystems, promotion of investment and competitiveness and, above all, to drive innovation, which depends crucially on the advance of productivity. Many maths applications are increasingly incorporated into capital goods. In recent years there has been a sharp reduction in the price of these goods (especially those relating to information and communication), facilitates their widespread use even by that part of the population with no knowledge of advanced mathematics. This last phenomenon could be facilitating more rapid dissemination of technology and, ultimately, higher growth in developing economies.

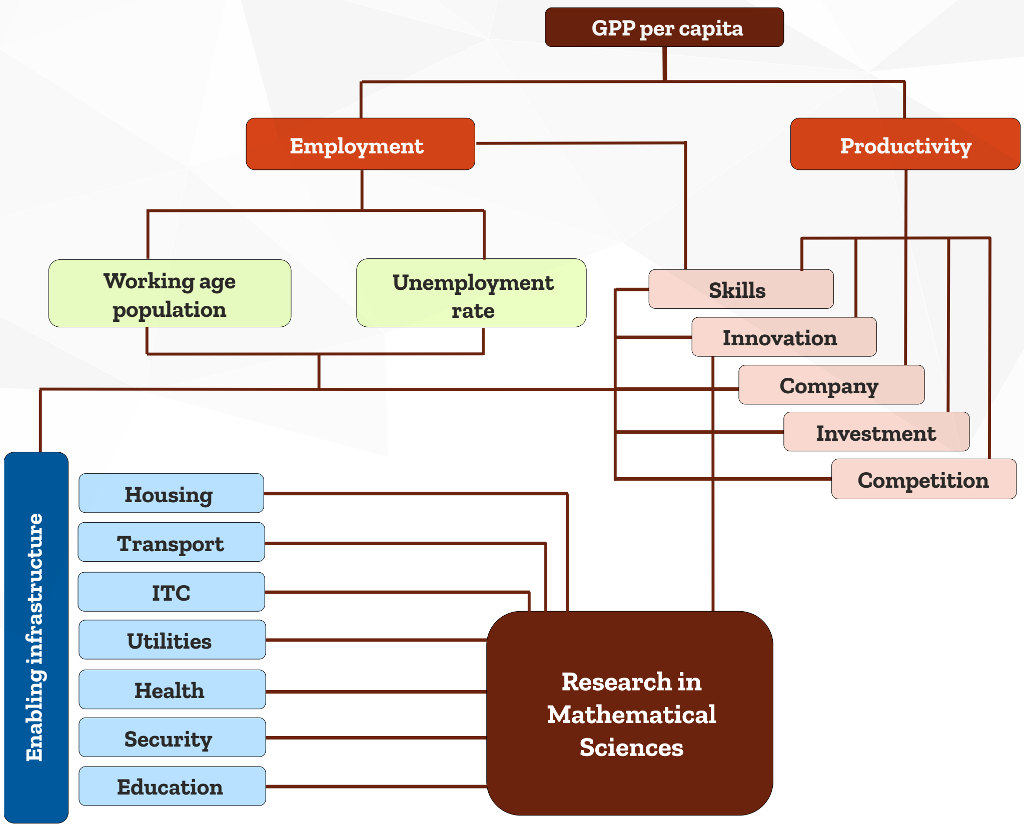
The mathematical sciences thus constitute a strategic knowledge for the Spanish economy and its business sector can take advantage of the opportunities arising from the technological revolution associated with robotisation and artificial intelligence (hereinafter, AI). Today it is a cornerstone of human capital formation, innovation and leadership in the field of digital transformation. Aghion, Jones and Jones (2017) analyse the impact that artificial intelligence could have on the long-term growth of economies, concluding that the positive effects could be enormous.

Artificial intelligence, a maths-intensive form of knowledge, will make it possible to achieve a new stage in the automation of tasks that will alleviate the negative effect of the ageing of the population on the long-term growth rate.

AI will make it possible to achieve a new stage in the automation of tasks, something that has characterised the different phases of technological progress since the advent of the industrial revolution. If machines can be made to replicate some forms of human intelligence, it will be possible to automate not only routine tasks but tasks that now require higher qualifications, boosting the growth of productivity and living standards. This transformation can also be applied to the production of knowledge itself, reducing the cost of the research. The combination of the two effects could change the recent trajectory of long-term growth and offset, in large part, the negative effect that the ageing of the population may have on the growth rate in many developed countries[13] (including Spain) over the long term.

13 The German government has launched a €3 billion investment plan for the development of AI, precisely in order to respond to the challenges of economic, social and demographic factors. This investment plan envisages the creation of 100 university chairs and the consolidation of a network of 12 research centres, focusing on the development and application of AI technologies.

Diagram **5. Long-term relationship between mathematical research and welfare**



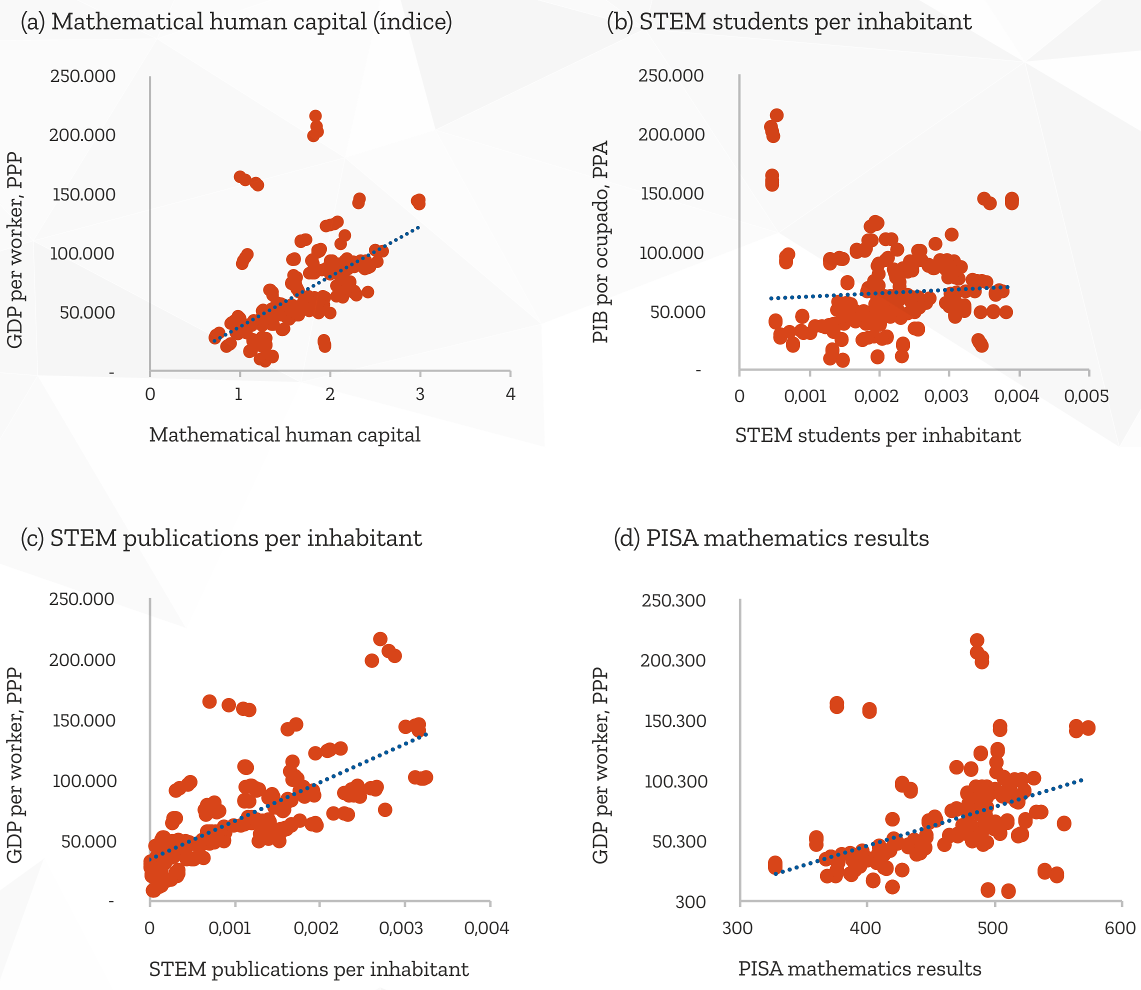
Source: Afi

5. 1. DYNAMIC EFFECTS OF MATHEMATICS IN THE SPANISH ECONOMY

This section presents a quantitative estimate of the impact of human mathematical capital on labour productivity in Spain in dynamic terms. The mere observation of positive relationships between certain relevant indicators associated with countries' mathematical research (results of evaluation in mathematics, number of publications cited in STEM disciplines[14] and proportion of graduates in these disciplines in the total population) and their labour productivity is suggestive of its importance for the performance of the latter.

14 STEM is the acronym for the terms Science, Technology, Engineering and Mathematics. For this exercise, account was taken of publications in the branches of mathematics, engineering, computer science, physics, astronomy, biochemistry and energy.

***Chart 11.* GDP per employee (in US$, PPP) and indicators associated with mathematical research, by country: (a) mathematical human capital, (b) STEM students per capita, (c) STEM publications per inhabitant and (d) PISA results in mathematics, 2013-2016**



Sources: Afi, OCDE, SCIMAGO, World Bank, UNESCO

Table 10 shows the output of different linear regressions (four, one for each column) through an estimation using Ordinary Least Squares (OLS). The main objective of a linear regression is to identify the effect that some specific variables (explanatory or independent, X_i) have over another different variable (dependent, Y). In this case, the aim is to explore the role of Mathematics on GDP per worker (labour productivity) on each country.

The economic variables used on this exercise, like GDP per worker or the stock of capital per worker, are publicly available on the World Bank database; this guarantees homogeneity in the estimation of these macro-magnitudes, and comparability between countries. However, those variables that reflect the level of mathematical knowledge in each country are less evident. For this case, the following parameters have been selected:

Table 10. **Relationship between the human capital index and its components, and GDP per employed worker. OLS estimation, 2013-2016**

	GDP per employed worker (in logarithm-logs)			
Mathematical human capital (logs)	0.323*** (0,071)			
STEM students (logs)		0.073** (0,037)		
STEM publications (logs)			0.070*** (0,016)	
PISA maths results (logs)				0.565*** (0,205)
Capital Stock per employed worker (logs)	0.717*** (0,025)	0.762*** (0,023)	0.672*** (0,029)	0.723*** (0,029)
Mediterranean Europe	-	-	-	-
Central Europe	0.131** (0,053)	0.181*** (0,052)	0.168*** (0,049)	0.147*** (0,052)
Europe Scandinavia	0.078 (0,052)	0.144*** (0,046)	0.121** (0,047)	0.132*** (0,046)
Eastern Europe	0.108* (0.051)	0.115** (0,054)	0.110** (0,055)	0.106* (0,055)
Anglo-Saxon countries	0.200*** (0,052)	0.241*** (0,049)	0.246*** (0,048)	0.244*** (0,048)
Latin America	0.254*** (0,060)	0.189*** (0,059)	0.243*** (0,065)	0.216*** (0,060)
Middle East	0.320*** (0,066)	0.275*** (0,065)	0.291** (0,066)	0.321*** (0,065)
East Asia	-0.002 (0,058)	0.015 (0,061)	0.041 (0,061)	-0.025 (0,066)
Year dummy	Sí	Sí	Sí	Sí
Countries	66	66	66	66
Observations	264	264	264	264
R^2	0,897	0,890	0,894	0,892

Note: The coefficients are expressed in logarithms. Typical robust standard deviations are shown in parentheses. *, **, *** Indicate statistical significance of 10%, 5% and 1% respectively.

Sources: Afi , OCDE, SCIMAGO, World Bank, UNESCO

If Spain increased the proportion of STEM graduates in the total population to the same level as France, labour productivity could increase by 2.2% over current levels.

- Number of students in STEM (Science, Technology, Engineering and Mathematics) degrees per inhabitant.
- Number of academic publications in a STEM classification per inhabitant.
- PISA (Programme for International Student Assessment) results in Mathematics for the country in question.

Using a panel data analysis methodology[15] for a total of 66 countries over the 2013-2016 period, these three indicators are estimated to have significant positive effects on worker productivity after adjusting for other factors, such as the level of physical capital per worker (see Table 10).

Although the estimates made with the three indicators provide similar readings, perhaps the variable that best approximates the level of mathematical human capital is the proportion of the population with STEM education (STEM students). This is because it is related to the population's stock of mathematical knowledge, to a greater extent than the quality of primary education or the results of academic research.

Thus, a simple exercise of convergence, in which Spain reaches the same stock of mathematical human capital as France, suggests the remarkable increases in labour productivity that might arise. In particular, if Spain increased the proportion of STEM graduates in the total population to the same level as France, labour productivity could increase by 2.2% over the current levels.

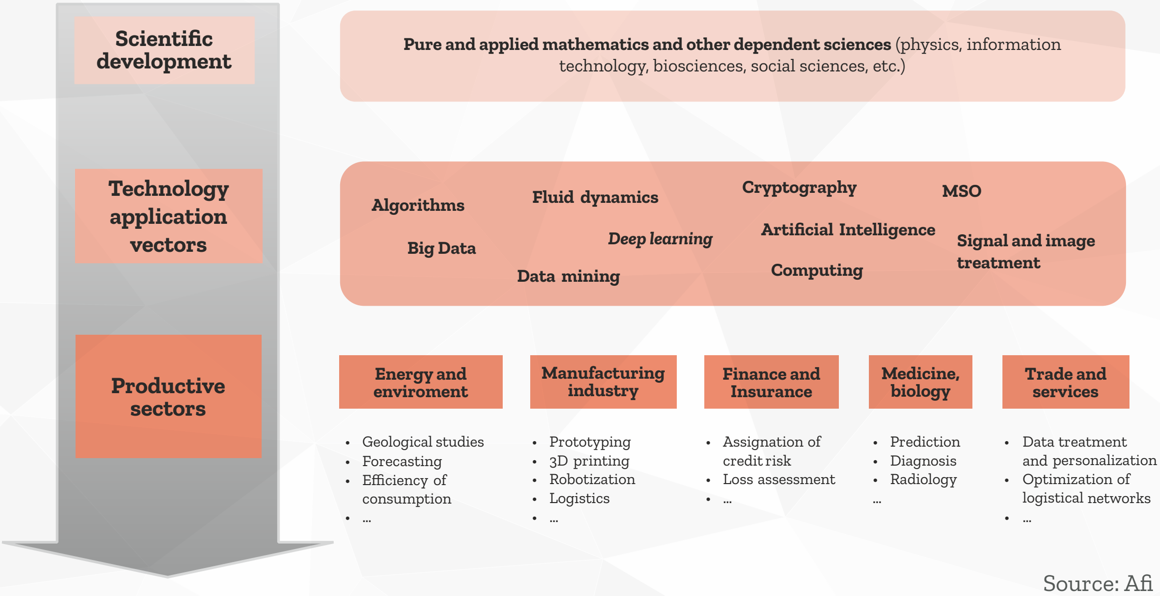
15 For more information, see Methodological Appendix 7.6 Impact estimate with panel data methodology.

5. 2. TRANSFER OF MATHEMATICAL TECHNOLOGY TO THE ECONOMY AND ASSOCIATED POSITIVE EXTERNALITIES. SOME EXAMPLES IN SPAIN

One of the peculiarities of the transfer of mathematical technology to productive activities is that its contribution, which is decisive, goes unnoticed because it often underlies what is actually observable. In the developed world, societies currently live in a super-technological reality, where everyday tools work using algorithms or other maths applications. By way of example, a GPS signal, often used to find an unknown destination via a mobile device, is a trigonometric calculation using information from satellites that identifies the exact geographical location of people and objects. In addition, communications in the form of mobile phone messages are digital signals converted back to numbers, but this reality is not perceived by the user as such.

The process of technological transfer starts with the interweaving between the more abstract mathematical knowledge incorporated in its vectors of application to the specific problems of each economic sector (see diagram 6). This process of transferring starts with the idea that any phenomenon or process being studied may be represented by mathematics. The application of mathematics starts with the observation of specific problems, for which a modelling or abstraction is carried out, which obviously varies greatly depending on the scope, complexity and sophistication of each case.

Diagram 6. Transfer of mathematical science and its application to economic sectors



Source: Afi

Once a model has been developed, the task of mathematicians is to understand its implications, and to establish critical judgements on its adaptation. To do this, they increasingly turn to computing tools in combination with statistical maths, which serve to validate such models through the use of actual data or simulations in specific scenarios of the phenomenon under investigation. In some cases, these models need to be improved in the light of the validation and the process can iterate between the comparison and the observation and development of subsequent models, whose ultimate aim is to achieve predictive power over the phenomenon in question.

In Spain, the transfer of mathematical technology is a growing phenomenon due to the creation of multi-disciplinary groups and the incorporation of workers with mathematical profiles to very diverse problems.

In Spain, the transfer of mathematical technology is a growing phenomenon, extending in the last decade to a diversity of sectors, due to the creation of multidisciplinary groups and the incorporation of workers with mathematical profiles to very diverse problems. However, there is still a long way to go to achieve the optimal level of linkage between maths and the real world. The panel of respondents (see appendix) coincides in pointing out two aspects of the analysis of the obstacles or challenges that must be faced by Spain to boost the transfer of mathematics:

- The need to improve the linkages between the mathematics training model and the needs of the economy. The syllabuses, from basic education to university, are not sufficiently adapted to the demands of the environment, in both technological centres and private companies themselves. Although education in mathematical sciences provides the capability to analyse, formulate problems and develop mathematical models, these skills are not sufficient for success in the professional field, as they create distort the aim of a greater mathematical presence in productive activities in two ways:

- Starting from basic education, the applicability of mathematics to real problems is not sufficiently instilled, and graduates themselves are wary of career alternatives to teaching or research.
- A large part of those university mathematics graduates who do not wish to engage in teaching or research are unaware of the enormous potential demand for mathematicians in diverse disciplines and economic activities.

- The obstacles to the insertion of mathematicians into the economy arise partly from ignorance of the value offered by mathematical human capital in the solution of organisational problems, improvement of efficiency, and business development, and, on the other hand, from the small average size of enterprises, a limiting factor inherent in the Spanish economy.

Here are some cases of success in mathematics transfer in Spain in specific sectors.

Success Case 1. Mathematics in the energy sector

Almost 50% of the energy sector's contribution to GVA is attributable to the input of mathematics. In fact, such an apparently striking result is not surprising. This is because the various activities that make up the energy sector have a strong demand for mathematics, given the high degree of capitalisation. Indeed, mathematics is present in virtually all phases of the value chain of this economic sector, from studies prior to the exploratory drilling for deposits, to those which seek to estimate the reliability of the duration of an electrical network. In addition, mathematical modelling enables the formulation of predictions for electricity consumption and fraud detection, as well as optimising generating assets or energy transport networks.

Success Case 2. Mathematics in the financial, auditing and insurance sectors

Mathematics is the most powerful tool available to humans to deal with decision-making in contexts of uncertainty. This is the essential reason why mathematics has such a strong presence in the financial and insurance sector.

In particular, in the field of insurance, the product traded between an insurer and insured is an insurance premium, which in reality reflects an actuarial or probabilistic calculation. In addition, algorithms are a tool to help with the assessment in cases of loss. Thus, close to 60% of the insurance sector's GVA corresponds to the mathematical input. This is similar to the percentage that it represents in the financial sector, closely linked to the latter, inasmuch as its activity is dependent on both modelling and risk management, and the intensive use of various capitalisation formulas. The massive availability of information in real time that has facilitated the latest technological developments is a huge field of experimentation for the introduction of mathematics in various aspects of financial activity, such as prediction of the demand for cash in ATMS.

Success Case 3. Mathematics in the biomedical sector

Perhaps one of the most relevant areas of transfer, due to its clear positive externalities for society, is in the biomedical field.

In Spain, various initiatives of enormous scope have been developed in the last few years. These include the design and implementation of markers for the early detection of Alzheimer's disease, the creation of probabilistic models for cancer diagnosis and the modelling of the cellular metabolism. On the other hand, mathematical tools have served to increase the sophistication of various treatments, such as the prosthesis adapted to the structure of a given patient or robotics in traumatological procedures, etc. Other noteworthy developments are the 3D reconstruction of organs, the application of algorithms for the classification of biological samples or logistical optimisation models in hospitals in regions with widely dispersed populations, as is the case of Castile and Leon.

Success Case 4. Mathematics in the environment

Another, increasingly important, area of mathematical application with generation of positive externalities is the environment. This comes, moreover, in a context marked by the urgency of policies to slow climate change trends that threaten the balance and sustainability of ecosystems. In Spain, a country that boasts a huge natural heritage, in recent decades mathematical transfer initiatives have proliferated to mitigate the effects of environmental problems, with models for the prediction of drought, floods or various adverse atmospheric phenomena. Also notable is the application of models of the risk of fire, one of the environmental problems that tend to hit certain areas of the peninsula on a regular basis.

6. CONCLUSIONS AND RECOMMENDATIONS FOR PUBLIC POLICY

The notable conclusions of this study include the following:

— The economic nature of mathematics is complex. It combines characteristics of a public club good with the status of a private good, due to the appropriable benefits arising from its use in economic activities.

— The role of mathematics in productive activities is cross-cutting, as it affects all sectors, as well as intervening in the different phases of the production process, from the design, modelling, simulation and prototyping of products to the optimisation of production processes and the organisation and analysis of data.

— The internet revolution has turned mathematics into a fundamental input of production, inasmuch as services have increasingly been incorporating both maths-based physical/technological capital (information and communications technologies, software, electronic devices, etc.), and mathematical human capital.

— Maths-intensive activities accounted for a million jobs in 2016, which represented 6% of total employment in the Spanish economy. Adding in the indirect and induced effects, the impact on employment would rise to 19.4% of the total.

— In terms of GVA, the impact of maths-intensive activities stood at 10.1% of the total in 2016 (26.9% of the total including knock-on effects).

— The impact of mathematics is greatest in the IT, finance, telecommunications services, and electricity and gas sectors.

— These impacts are smaller than those calculated for other European countries where similar studies have been carried out; there, the direct impact in terms of employment ranges between 10% and 11% of the total, whereas in GVA the interval is 13-16% of the total.

— The difference is due largely to the Spanish economic structure, more oriented toward activities with a lower presence of maths-intensive professions.

— Maths is a basic component of higher labour productivity, constituting a strategic knowledge for the Spanish economy which would enable its business sector to take advantage of the opportunities arising from the technological revolution associated with robots and artificial intelligence.

— If Spain increased the proportion of STEM graduates in the total population to the same level as France, labour productivity could increase by 2.2% over current levels.

For all these reasons, it would be advisable to:

— Rethink the educational model to (i) increase the presence of mathematics in educational programmes and (ii) increase mathematicians' understanding of the applicability in the real world of the tools and skills acquired in their education.

— Improve the linkages between the mathematics training model and the needs of the economy, bringing the University and Research Centres closer to companies through meetings, cooperation agreements, work experience for students, etc.

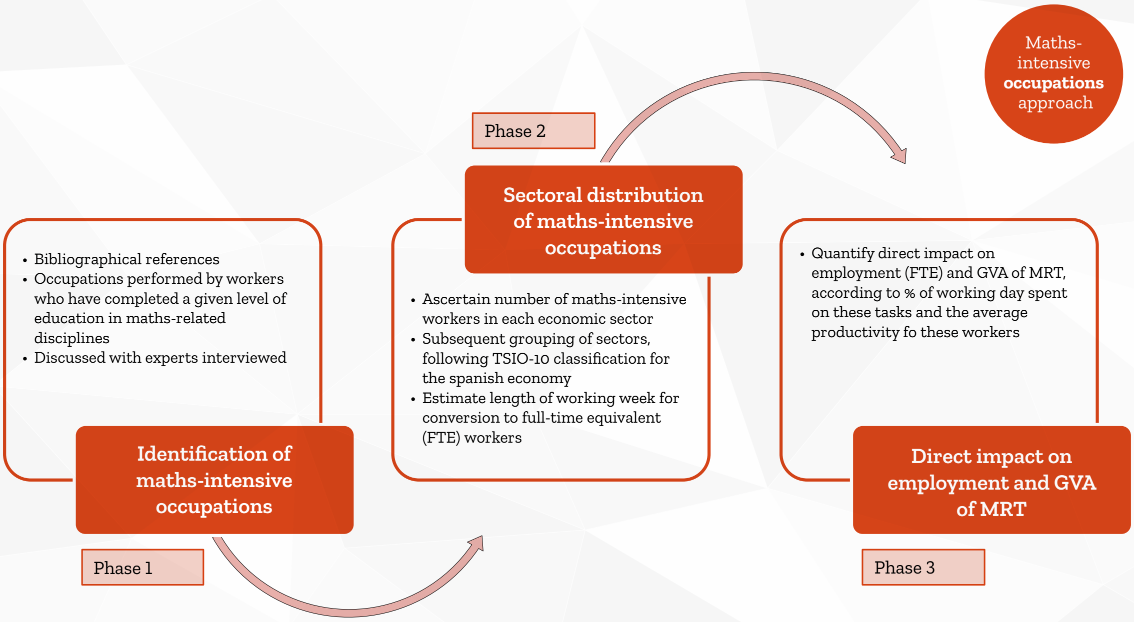
— Boost R&D&i spending in the mathematical sciences, since clear externalities derive from both the STEM sciences and from the vectors of their technological application to the economy, leading to welfare gains for society as a whole.

— Increase the incentives for applied mathematics, seeking to match those offered to scientific research, so that progress is transferred to the economy.

— Ensure that the utility resulting from the incorporation of mathematicians in the various phases of the productive process is visible in the business environment, as well as in all types of organisations, by holding meetings between universities and business organisations, sharing success stories, etc.

7. METHODOLOGICAL APPENDIX

Diagram 7. Methodology used to estimate the importance of MRT in the Spanish economy through the occupations approach



Source: Afi

7. 1. OCCUPATIONS APPROACH: METHODOLOGY

The maths-intensive occupations approach will consist of three distinct phases (see diagram 7):

Phase 1: Identification of maths-intensive occupations

The objective of this phase is to detect the professions engaged in mathematical research and/or making intensive use of mathematical tools. To do this, three complementary methodologies will be used:

— Bibliographical references.
— Use of the EPA (INE) microdata.
— Discussions with experts in the transfer of mathematics.

Bibliographical references

This methodology consists in comparing the selection of maths-intensive occupations in the studies developed in other countries with the corresponding occupations in Spain's 2011 National Classification of Occupations.

As indicated above, the documentation consulted notably includes studies carried out in the UK, the Netherlands and France. However, the study serving as a frame of reference was that of the UK, for three reasons:

1. **It has more methodological detail** (the occupations, shown in the methodological appendix of this report, have a breakdown level of four digits) with which to contrast the 170 categories of the Spain's CNO-11 used here (EPA microdata with detail to three digits).

2. **It quantifies the existing employment** (in absolute and relative terms) in MRT occupations and identifies the proportion of working hours actually dedicated to mathematical research or the intensive use of mathematical tools.

3. **It was the first study published**, meaning that in one way or another it has influenced the methodology used in the rest of studies consulted for this report (as recognised in the Netherlands and France reports).

Given that the classification of occupations in Spain is different to that of the UK, and there is no official table of equivalence between the two classifications, an exercise in comparison was conducted, identifying those with a certain similarity[16].

It was concluded Spain has 37 maths-intensive occupations (out of a total of 170, i.e. 21% of the total), as there are parallels with studies conducted in other countries.

Use of the EPA (INE) microdata

In addition to the analysis of the recent literature, those occupations performed by Spanish workers requiring maths skills were identified using the statistical information available.

The aim of this phase was to quantify the number of workers who have completed a certain level of education in maths-related academic disciplines in each of the occupational categories available. Account should be taken of the following considerations:

— On the one hand, what is meant by maths-related disciplines?

— On the other hand, what level of education completed by the worker should be considered sufficient to perform maths-intensive tasks?

The Labour Force Survey (EPA) microdata provided by the INE for 2016 (the latest available) enable each type of occupation in the CNO-11 to three digits (170 types) to be cross-referenced with the following variables:

- **Area of the level of studies and/or academic discipline.** Firstly, from the classification of programmes, qualifications and certifications in areas of study (CNED-F-14), the maths-related academic disciplines were selected, since workers with this type of education will have maths skills to be used in the workplace. That is to say, it is assumed that workers with this type of education practice the profession for which they have been trained. 10 of the 40 disciplines that can be identified from the EPA microdata were considered. These are:
    - Social and behavioural sciences.
    - Business and management.
    - Biology and biochemistry.
    - Environment, natural environments and wildlife.
    - Chemical, physical and geological sciences.
    - Mathematics and statistics.
    - Information and communication technologies (ICT).
    - Mechanics, electronics and other technical training.
    - Construction.
    - Health.

16 For more details, see 7.2. Occupations approach: bibliographical references.

- **Level of training completed:** from the above, on this occasion it was necessary to classify the jobs according to the level of training they have completed:

    - Low: comprising illiterate workers and those who have completed only primary education. In this case, maths skills are assumed to be basic; therefore, this type of worker would not be sufficiently qualified to perform maths-intensive tasks.
    - Medium: includes workers who have completed secondary school or vocational training course (intermediate level or higher). Technical or support occupations may be performed by workers who have completed this level of training; therefore, they will be taken into account for the purpose of estimating jobs with MRT.
    - High: includes workers who have completed at least one university degree (including also those who have done a master or doctorate). This type of workers are perhaps the most qualified to perform occupations with MRT.

    The results shown in this estimation exercise can admit different aggregates of jobs, according to the level of studies that have been completed. The first aggregate refers to those professionals who have higher education, i.e. they have a university degree. Subsequently, those workers who had completed some degree of vocational training (upper and middle, in that order) were added, together with those who had completed secondary education, as they could also be performing tasks of certain mathematical intensity.

To quantify the workers with maths skills in each occupation it is necessary to take account of a phenomenon of growing importance in the Spanish economy: overqualification. Since the start of the economic and employment recovery (in 2014), the most skilled workers who were unemployed returned to the labour market, taking jobs for which they were overqualified. The latest estimates[17] suggest that two out of every three new jobs were overqualified. This phenomenon means there may be workers with maths skills in occupations that are not really maths-intensive and, therefore, do not make use of such skills. It is therefore necessary to establish a threshold of workers with maths skills in each profession that reflects the real need for workers to have this type of skills.

Assuming that maths-intensive occupations are those in which at least 70% of workers have maths skills, it can be concluded that Spain would have between 15-20 maths-intensive occupations, according to the level of training completed by these professionals (out of a total of 170, i.e. between 8-12% of the total)[18].

However, a complementary analysis is required, because many disciplines are involved (e.g. construction), with very different professional categories (from architects and engineers, who are relevant to this report, to machine operators and labourers, who are not).

Combining the results of the two methodologies employed (bibliography and EPA), there would be 40 maths-related occupations (see Table 11), out of a total of 170, or 23% of the total available categories.

17 Afi-ASEMPLEO SLM Monitor, No. 115, February 2017.

18 For more information, see 7.3. Occupations approach: use of EPA microdata.

Table 11. List of maths-intensive occupations according to CNO-11 (3 digits)

Code	Description of the occupations	Level of education completed			
		High	High + upper vocational	High + upper vocational + middle vocational	High + upper vocational + middle voc. + secondary
111	Members of the executive branch and legislative bodies; managers of the public administration and organisations of social interest				
112	General Managers and executive presidents				
121	Directors of administrative departments				
122	Managers of sales, advertising, public relations and research and development				
132	Directors of information and communications technologies (ICT) and professional service firms				
143	Directors and managers of wholesale and retail trade companies				
211	Physicians				
212	Nursing and midwives				
214	Pharmacists				
215	Other health professionals				
241	Physicists, chemists, mathematicians, and related fields				
243	Engineers (except agricultural, forestry, electrical, electronic and ICT engineers)				
244	Electrical, electronic and telecommunications engineers				
245	Architects, urban planners and geographical engineers				
246	Non-graduate engineers (except agricultural, forest, electrical, electronic and ICT engineers)				
247	Non-graduate engineers in electricity, electronics and telecommunications				
248	Non-graduate architects, surveyors and designers				
261	Finance specialists				
262	Organisation and management specialists				
264	Technical and medical sales professionals (except ICT)				
271	Software and multimedia analysts and designers				
272	Database and network specialists				
281	Economists				
311	Technical draughtsmen				
312	Technicians in physics, chemistry, and environmental and engineering sciences				
313	Process control technicians				
315	Professionals in maritime and aeronautical navigation				
316	Quality control technicians of the physical, chemical and engineering sciences				
320	Mining, manufacturing and construction engineering supervisors				
331	Laboratory, diagnostic testing and prostheses technicians				
340	Support Professionals in finance and mathematics				
361	Administrative and specialised assistants				
362	Customs and tax agents and similar working in tasks typical of Public Administration				
381	Information technology operations and user assistance technicians				
382	Computer programmers				
383	Audiovisual recording, broadcasting and telecommunications technicians				
411	Accounting and finance employees				
561	Nursing auxiliaries				
562	Auxiliary Technicians in pharmacy and health emergencies and other carers of persons in health services				
752	Other installation and electrical equipment technicians				
	Total	**39**	**39**	**39**	**40**

Source: Afi, INE (EPA microdata)

Discussions with experts in the transfer of mathematics

This methodology is based on a selection of professionals dedicated to mathematical research and the intensive use of mathematical tools, who have extensive experience in the transfer of mathematics to different economic activities in Spain[19]. With the results of the previous methodologies, the maths-intensive occupations identified were discussed with the experts consulted.

The interviews with experts in mathematics transfer corroborate the finding that the occupations listed in Table 11 are those that are related in one way or another with the intensity of mathematics. Nevertheless, they attribute more relevance to the share of the working day spent making use of maths skills than to the fact that these are eminently mathematical professions. This aspect will be taken into account in phase 2 of the quantitative estimate of direct employment.

Phase 2: Sectoral distribution of maths-intensive occupations

The aim of this phase is to learn the penetration of this type of worker in the Spanish economy. It is necessary to perform the following tasks:

— **Detect the number of maths-intensive workers** in each sector of the economy.

— **Sectoral grouping according to the 2010 symmetrical input-output tables (TSIO-10) classification** of the Spanish economy, since the ultimate objective is to calculate the knock-on effects (indirect and induced) that they exert on other economic activities, for which the methodology of the Input-Output Tables is used.

— **Estimate the length of the working week** to convert the workers in each sector to full-time equivalents (FTE), the measure most often used to quantify the direct impact of an activity in terms of employment.

The EPA microdata enable each of the 40 occupations identified to be cross-referenced with the following variables:

- **Economic sector:** distribution of the maths-intensive workers in each sector.

    - The EPA microdata have 272 possible classifications according to the 3 digit CNAE-09.
    - However, for the calculation of the economic impact, it is necessary to take into account the TSIO-10 classification, the basis for the input-output tables for the Spanish economy. According to this other classification of economic activities, there would be a total of 63 different sectors.
    - Therefore, we have aggregated the three-digit sectors according to the equivalence between the CNAE-09 and the TSIO-10 provided by the INE[20].

- **Length of working week:** the microdata enable the number of hours worked per week per worker to be estimated.

    - The aim is to calculate the full-time equivalent workers (those who work 40

19 For more detail on the experts consulted, see 8. Experts consulted.

20 For more information, please see the following link: http://www.ine.es/daco/daco42/cne10/cne_tio_10.xlsx

hours a week), the variable of direct impact on employment that we seek to estimate.

- Workers exercising a profession and with an average working weekly of more than 40 hours become full-time equivalents through the removal of the overtime, which is integrated with the other workers of the same profession, in such a way that it is transformed into fictitious workers. Thus, the number of jobs in this profession is higher than that reflected in the EPA. The opposite is true if the average working weekly is less than 40 hours.
- The following assumption was used in developing these estimates: it has been considered that all workers (self-employed or salaried employees) have a permanent contract, i.e. they work during the 52 weeks of the year, for the purposes of calculating the direct impact of MRT activities on an annual basis.

Phase 3: Direct impact on employment and GVA of maths-intensive activities

The aim of this phase is to calculate the direct impact of MRT on employment and GVA in the Spanish economy, through the quantification of the employment (FTE) and GVA of the MRT, according to the proportion of the working day dedicated to these tasks and the average productivity of these workers in the economic sector concerned. Both figures are derived from the EPA microdata and the Input-Output Tables for the Spanish economy, as indicated earlier.

7. 2. OCCUPATIONS APPROACH: BIBLIOGRAPHICAL REFERENCES

Comparison with classification of occupations in the United Kingdom

One of the appendixes of the study conducted by Deloitte in the UK has a list of tables (see Tables 12 and 13) which can be used to identify occupations of certain mathematical intensity, as well as the proportion of working time associated with each of the occupations, as the quantification of the workers is calculated on a full-time equivalent basis. The classification follows a standardised typology according to the UK SOC, with detail to four digits.

The allocation of maths-intensive workers and hours is based on four criteria:

- Transfer of the mathematical sciences to each economic sector and professional occupation.
- The author's interpretation with regard to the incorporation of graduates in mathematics and other academic disciplines with high mathematical content into the economy.
- A review of the literature.
- The opinions expressed by experts in mathematics transfer who were also consulted for this report.

Table 12. Workers by maths-intensive occupation in the UK (number and % total), 1/2

Level 4 SOC code	Include entire occupation as mathematics occupation?	Apportion needed?	% of category included	Final number of mathematical science occupations	Total number of jobs in SOC category
1111 Senior officials in national government	N	Y	*	400	*
1112 Directors & chief execs of major organisations	N	Y	5%	2,800	60,200
1113 Senior officials in local government	N	Y	5%	1,600	35,100
1121 Prod. works & maintenance managers	N	Y	5%	19,300	413,500
1123 Managers in mining and energy	N	Y	*	12,600	*
1131 Financial managers & chartered secretaries	N	Y	5%	12,100	259,200
1132 Marketing and sales managers	N	Y	5%	25,600	549,400
1133 Purchasing managers	N	Y	5%	2,300	50,100
1136 Information & communication technology managers	Y	N	na	309,900	309,900
1137 Research and development managers	Y	N	na	51,500	51,500
1141 Quality assurance managers	Y	N	na	46,600	46,600
1151 Financial institution managers	N	Y	5%	7,200	154,100
1181 Hospital and health service managers	Y	N	na	77,000	77,000
2111 Chemists	Y	N	na	27,600	27,600
2112 Bio scientists and biochemists	Y	N	na	94,200	94,200
2113 Physicists, geologists & meteorologists	Y	N	na	26,100	26,100
2121 Civil engineers	Y	N	na	78,700	78,700
2122 Mechanical engineers	Y	N	na	78,300	78,300
2123 Electrical engineers	Y	N	na	59,800	59,800
2124 Electronics engineers	Y	N	na	35,900	35,900
2125 Chemical engineers	Y	N	na	9,500	9,500
2126 Design and development engineers	Y	N	na	63,300	63,300
2127 Production and process engineers	Y	N	na	31,200	31,200
2128 Planning and quality control engineers	Y	N	na	29,200	29,200
2129 Engineering professionals not elsewhere classified	Y	N	na	97,100	97,100
2131 IT strategy and planning professionals	Y	N	na	48,900	148,900
2132 Software professionals	Y	N	na	327,500	327,500
2211 Medical practitioners	N	Y	10%	24,200	242,900
2212 Psychologists	N	Y	10%	3,000	29,900
2213 Pharmacists & pharmacologists	Y	N	na	41,800	41,800
2215 Dental practitioners	N	N	na	0	35,700
2311 Higher education teaching professionals	N	Y	10%	13,200	132,600
2317 Registrars & senior administrators of educational establishments	N	Y	10%	4,700	47,200
2321 Scientific researchers	Y	N	na	17,100	17,100
2322 Social science researchers	Y	N	na	17,100	17,100
2329 Researchers not elsewhere classified	Y	N	na	49,100	49,100
2421 Chartered and certified accountants	N	Y	10%	15,900	160,000
2422 Management accountants	Y	N	na	85,700	85,700
2423 Management consultants, actuaries, economists & statisticians	Y	N	na	180,400	180,400
2431 Architects	N	N	na	0	55,400
2432 Town planners	N	Y	10%	2,000	20,700
2433 Quantity surveyors	N	N	0%	0	39,700

Source: Deloitte

Table 13. Workers by maths-intensive occupation in the UK (number and % total), 2/2

Level 4 SOC code	Include entire occupation as mathematics occupation?	Apportion needed?	% of category included	Final number of mathematical science occupations	Total number of jobs in SOC category
2434 Chartered surveyors (not quantity surveyors)	N	N	na	0	63,500
2441 Public service administrative professionals	Y	N	na	34,900	34,900
3111 Laboratory technicians	Y	N	na	64,900	64,900
3112 Electrical & electronic technicians	Y	N	na	26,500	26,500
3113 Engineering technicians	Y	N	na	73,500	73,500
3114 Build & civil eng technicians	Y	N	na	25,800	25,800
3115 Quality assurance technicians	Y	N	na	15,700	15,700
3119 Science & engineering technicians not elsewhere classified	Y	N	na	38,200	38,200
3131 IT operations technicians	N	N	na	0	117,000
3132 IT user support technicians	N	N	na	0	65,400
3211 Nurses	N	N	na	0	509,300
3212 Midwives	N	N	na	0	35,100
3213 Paramedics	N	N	na	0	20,800
3511 Air traffic controllers	Y	N	*	8,900	*
3512 Aircraft pilots and flight engineers	Y	N	na	22,600	22,600
3513 Ship and hovercraft officers	N	N	na	0	15,200
3531 Estimators, valuers and assessors	N	N	na	0	55,300
3532 Brokers	N	Y	0%	0	50,900
3533 Insurance underwriters	Y	N	na	30,100	30,100
3534 Fin. & invest. analyst & advisers	Y	N	na	173,900	173,900
3535 Taxation experts	N	Y	4%	900	21,500
3537 Financial and accounting technicians	N	N	na	0	30,800
3568 Environmental health officers	Y	N	4%	11,900	11,900
4121 Credit controllers	Y	N	4%	45,600	45,600
4122 Accounts wages clerk, bookkeeper	N	N	na	0	512,700
5242 Telecommunications engineers	Y	N	na	42,600	42,600
5245 Comp engineer, installation & maintenance	N	N	na	0	43,500
5249 Electrical & electronics engineer not elsewhere classified	Y	N	na	83,500	83,500

Source: Deloitte using ONS data. Numbers are rounded. * refer to where the underlying data is not available publicly and hence the adjustment percentage cannot be disclosed. Note, numbers do not sum to 2.8 million due to rounding.

Source: Deloitte

On the basis of this selection and the Spanish National Classification of Occupations 2011, 37 maths-intensive occupations have been identified, representing 20% of the total categories (170 different types of occupations). The numerical codes and their descriptions are as follows:

111 Members of the executive branch and legislative bodies; managers of the public administration and organisations of social interest.

112 General Managers and executive presidents.

121 Directors of administrative departments.

122 Managers of sales, advertising, public relations and research and development.
132 Directors of information and communications technologies (ICT) and professional service firms.
143 Directors and managers of wholesale and retail trade companies.
211 Physicians.
212 Nursing and midwives.
214 Pharmacists.
241 Physicists, chemists, mathematicians, and related fields.
243 Engineers (except agricultural, forestry, electrical, electronic and ICT engineers).
244 Electrical, electronic and telecommunications engineers.
245 Architects, urban planners and geographical engineers.
246 Non-graduate engineers (except agricultural, forest, electrical, electronic and ICT engineers).
247 Non-graduate engineers in electricity, electronics and telecommunications.
248 Non-graduate architects, surveyors and designers.
261 Finance specialists.
262 Organisation and management specialists.
271 Software and multimedia analysts and designers.
272 Database and network specialists.
281 Economists.
311 Technical draughtsmen.
312 Technicians in physics, chemistry, and environmental and engineering sciences.
313 Process control technicians.
315 Professionals in maritime and aeronautical navigation.
316 Quality control technicians of the physical, chemical and engineering sciences.
320 Mining, manufacturing and construction engineering supervisors.
331 Laboratory, diagnostic testing and prostheses technicians.
340 Support Professionals in finance and mathematics.
361 Administrative and specialised assistants.
362 Customs and tax agents and similar working in tasks typical of Public Administration.
381 Information technology operations and user assistance technicians.
382 Computer programmers.
383 Audiovisual recording, broadcasting and telecommunications technicians.
411 Accounting and finance employees.
561 Nursing auxiliaries.
562 Auxiliary Technicians in pharmacy and health emergencies and other carers of persons in health services.

7. 3. OCCUPATIONS APPROACH: EPA MICRODATA

Identification of maths-intensive occupations from the EPA (INE) microdata

The analysis of the EPA microdata at the two digit level of the CNO-11 enables the identification of the level and the discipline of studies completed by each worker for each of the 170 existing types of occupation.

The tables below (see Tables 14, 15, 16 and 17) quantify, for each of the occupations, the number of workers with mathematical training according to the level of studies concerned, as the aim is to cover all maths-related groups and to assess the sensitivity of broadening the scope beyond university graduates.

Assuming that maths-intensive occupations will be those in which at least 70% of workers have maths skills, it can be concluded that Spain would have between 15-20 maths-intensive occupations (those shaded in tables 14, 15, 16 and 17), according to the level of education completed by these workers (out of a total of 170, i.e. between 8-12% of the total).

Table 14. Workers who have completed maths-related studies by occupation and level of maths achieved (number and % total), 1/4

Classification of occupations according to CNO-11 (3 digits)		Workers according to the level of studies concerned						Determination of maths-intensive occupations according to the level of education completed (accum.)			
		High	Middle				Total	High	High+ upper vocational	High+ upper vocational +middle vocational	High+ upper vocational +middle vocational +secondary
			Total	Secondary	Middle vocational	Upper vocational					
Code	Description	People						% total			
001	Officers and NCOs of the armed forces	3.755	3.082	329	0	2.752	29.476	12,7%	22,1%	22,1%	23,2%
002	Rank and file of the armed forces	270	10.508	5.723	0	4.785	53.154	0,5%	9,5%	9,5%	20,3%
111	Members of the executive branch and legislative bodies; managers of the public administration and organisations of social interest							0,0%	0,0%	0,0%	0,0%
112	General Managers and executive presidents	9.193	92	0	92	0	18.421	49,9%	49,9%	50,4%	50,4%
121	Directors of administrative departments	48.843	3.384	2.271	0	1.112	87.876	55,6%	56,8%	56,8%	59,4%
122	Managers of sales, advertising, public relations and research and development	43.396	7.512	1.236	0	6.276	82.473	52,6%	60,2%	60,2%	61,7%
131	Production managers in farming, forestry and fishing, and in manufacturing, mining, construction and distribution							0,0%	0,0%	0,0%	0,0%
132	Directors of information and communications technologies (ICT) and professional service firms							0,0%	0,0%	0,0%	0,0%
141	Directors and managers in the hotels sector	2.030	1.033	0	0	1.033	20.616	9,8%	14,9%	14,9%	14,9%
142	Directors and managers in the restaurants sector	8.276	1.583	536	0	1.047	57.768	14,3%	16,1%	16,1%	17,1%
143	Directors and managers of wholesale and retail trade companies	12.147	12.587	4.113	0	8.474	98.255	12,4%	21,0%	21,0%	25,2%
150	Directors and managers of companies in other service industries	20.937	12.314	4.396	0	7.918	91.964	22,8%	31,4%	31,4%	36,2%
211	Physicians	185.302	0	0	0	0	186.765	99,2%	99,2%	99,2%	99,2%
212	Nurses and midwives	262.437	1.877	0	0	1.877	271.451	96,7%	97,4%	97,4%	97,4%
213	Veterinarians	2.534	0	0	0	0	31.828	8,0%	8,0%	8,0%	8,0%
214	Pharmacists	68.025	0	0	0	0	69.248	98,2%	98,2%	98,2%	98,2%
215	Other health professionals	108.424	1.433	745	0	689	115.052	94,2%	94,8%	94,8%	95,5%
221	Lecturers in universities and other higher education (except vocational training)	50.051	0	0	0	0	83.861	59,7%	59,7%	59,7%	59,7%
222	Lecturers in vocational training	22.414	4.033	383	0	3.649	46.881	47,8%	55,6%	55,6%	56,4%
223	Secondary school teachers (except vocational training subjects)	87.475	217	0	0	217	281.446	31,1%	31,2%	31,2%	31,2%
224	Primary school teachers	12.291	0	0	0	0	256.216	4,8%	4,8%	4,8%	4,8%
225	Nursery school teachers	7.085	685	0	0	685	131.660	5,4%	5,9%	5,9%	5,9%
231	Special education teachers	6.207	2.270	819	0	1.451	36.174	17,2%	21,2%	21,2%	23,4%
232	Other teachers and teaching professionals	39.902	6.113	261	0	5.852	160.580	24,8%	28,5%	28,5%	28,7%
241	Physicists, chemists, mathematicians, and related	29.041	0	0	0	0	31.553	92,0%	92,0%	92,0%	92,0%
242	Professionals in natural sciences	35.606	0	0	0	0	59.843	59,5%	59,5%	59,5%	59,5%
243	Engineers (except agricultural, forestry, electrical, electronic and ICT engineers)	85.815	0	0	0	0	118.785	72,2%	72,2%	72,2%	72,2%
244	Electrical, electronic and telecommunications engineers	45.429	0	0	0	0	48.053	94,5%	94,5%	94,5%	94,5%
245	Architects, urban planners and geogr. engineers	45.415	0	0	0	0	52.085	87,2%	87,2%	87,2%	87,2%
246	Non-graduate engineers (except agricultural, forest, electrical, electronic and ICT engineers)	52.052	0	0	0	0	60.664	85,8%	85,8%	85,8%	85,8%
247	Non-graduate engineers in electricity, electronics and telecommunications	20.855	0	0	0	0	25.844	80,7%	80,7%	80,7%	80,7%
248	Non-graduate archs., surveyors and designers	55.686	8.415	1.018	0	7.398	102.810	54,2%	61,4%	61,4%	62,3%
251	Judges, magistrates, lawyers and prosecutors	7.246	0	0	0	0	120.764	6,0%	6,0%	6,0%	6,0%
259	Other legal professionals	19.933	0	0	0	0	59.982	33,2%	33,2%	33,2%	33,2%
261	Finance specialists	79.070	313	0	0	313	89.310	88,5%	88,9%	88,9%	88,9%
262	Organisation and management specialists	106.106	2.828	0	0	2.828	200.191	53,0%	54,4%	54,4%	54,4%
263	Technicians in the tourism sector	648	0	0	0	0	1.578	41,1%	41,1%	41,1%	41,1%
264	Technical and medical sales professionals (except ITC)	15.068	1.499	0	0	1.499	19.847	75,9%	83,5%	83,5%	83,5%
265	Other sales, marketing, advertising and public relations professionals	36.356	4.517	0	0	4.517	75.944	47,9%	53,8%	53,8%	53,8%
271	Software and multimedia analysts and designers	97.431	3.953	0	0	3.953	109.665	88,8%	92,4%	92,4%	92,4%
272	Database and network specialists	36.663	3.647	0	0	3.647	40.728	90,0%	99,0%	99,0%	99,0%
281	Economists	45.599	0	0	0	0	46.215	98,7%	98,7%	98,7%	98,7%
282	Sociologists, historians, psychologists and other social sciences professionals	68.593	2.355	0	0	2.355	148.905	46,1%	47,6%	47,6%	47,6%

Sources: Afi, INE (EPA microdata)

Table 15. Workers who have completed maths-related studies by occupation and level of maths achieved (number and % total), 2/4

Classification of occupations according to CNO-11 (3 digits)		Workers according to the level of studies concerned						Determination of maths-intensive occupations according to the level of education completed (accum.)			
		High	Middle				Total	High	High+ upper vocatio-nal	High+ upper vocational +middle vocational	High+ upper vocational +middle vocational +secon-dary
			Total	Secon-dary	Middle vocatio-nal	Upper vocatio-nal					
Code	Description	People						% total			
283	Priests of all religions	801	0	0	0	0	8.764	9,1%	9,1%	9,1%	9,1%
291	Archivists, librarians, curators and similar	2.285	0	0	0	0	15.689	14,6%	14,6%	14,6%	14,6%
292	Writers, journalists and linguists	6.611	1.823	0	0	1.823	73.858	9,0%	11,4%	11,4%	11,4%
293	Creative and interpretative artists	637	1.405	623	0	782	43.234	1,5%	3,3%	3,3%	4,7%
311	Technical draughtsmen	3.458	24.204	2.122	0	22.082	42.574	8,1%	60,0%	60,0%	65,0%
312	Technicians in physics, chemistry, and environmental and engineering sciences	18.005	57.646	7.729	464	49.452	103.657	17,4%	65,1%	65,5%	73,0%
313	Process control technicians	2.197	16.854	5.311	0	11.543	39.995	5,5%	34,4%	34,4%	47,6%
314	Natural sciences technicians and similar auxiliary professionals	5.592	4.569	1.324	0	3.246	20.336	27,5%	43,5%	43,5%	50,0%
315	Professionals in maritime and aeronautical navigation	4.306	1.707	286	0	1.421	15.369	28,0%	37,3%	37,3%	39,1%
316	Quality control technicians of the physical, chemical and engineering sciences	8.234	20.571	5.439	0	15.132	55.150	14,9%	42,4%	42,4%	52,2%
320	Mining, manufacturing and construction engineering supervisors	11.610	23.581	4.073	0	19.509	98.251	11,8%	31,7%	31,7%	35,8%
331	Laboratory, diagnostic testing and prostheses technicians	1.114	42.078	2.459	0	39.619	49.247	2,3%	82,7%	82,7%	87,7%
332	Other healthcare technicians	13.733	29.224	4.255	0	24.969	65.816	20,9%	58,8%	58,8%	65,3%
333	Alternative therapy professionals	1.772	1.172	1.172	0	0	6.676	26,5%	26,5%	26,5%	44,1%
340	Support professionals in finance and mathematics	36.159	10.114	0	0	10.114	70.524	51,3%	65,6%	65,6%	65,6%
351	Commercial agents and representatives	55.892	87.673	30.364	0	57.309	413.575	13,5%	27,4%	27,4%	34,7%
352	Other commercial agents	17.642	25.624	7.971	0	17.653	104.010	17,0%	33,9%	33,9%	41,6%
353	Real estate and other agents	23.225	18.565	3.477	3.019	12.070	95.789	24,2%	36,8%	40,0%	43,6%
361	Administrative and specialised assistants	55.232	74.072	14.967	0	59.105	317.214	17,4%	36,0%	36,0%	40,8%
362	Customs and tax agents and similar working in tasks typical of Public Administration	1.884	0	0	0	0	11.127	16,9%	16,9%	16,9%	16,9%
363	Security agency technicians	228	253	0	0	253	10.798	2,1%	4,4%	4,4%	4,4%
371	Support professionals in legal and social services	3.548	11.010	3.088	0	7.922	53.165	6,7%	21,6%	21,6%	27,4%
372	Sportspersons, coaches, sporting instructors; recreational instructors	10.740	10.094	2.382	0	7.712	121.415	8,8%	15,2%	15,2%	17,2%
373	Support technicians and professionals in cultural, culinary and artistic activities	3.652	6.782	1.778	0	5.005	66.090	5,5%	13,1%	13,1%	15,8%
381	Information technology operations and user assistance technicians	15.080	49.940	6.132	0	43.808	95.268	15,8%	61,8%	61,8%	68,3%
382	Computer programmers	37.403	39.879	2.570	0	37.309	96.334	38,8%	77,6%	77,6%	80,2%
383	Audiovisual recording, broadcasting and telecommunications technicians	8.291	24.932	1.940	704	22.288	55.833	14,8%	54,8%	56,0%	59,5%
411	Accounting and finance employees	99.114	61.902	9.582	0	52.320	254.816	38,9%	59,4%	59,4%	63,2%
412	Employees in goods recording and production and transport support services	24.251	60.886	26.228	0	34.658	209.717	11,6%	28,1%	28,1%	40,6%
421	Library and archive employees	864	988	725	0	262	8.851	9,8%	12,7%	12,7%	20,9%
422	Employees of postal services, encoders, proofreaders and personnel services	4.264	18.468	7.863	0	10.605	82.638	5,2%	18,0%	18,0%	27,5%
430	Other clerical employees not attending the public	85.940	131.643	37.825	0	93.818	474.213	18,1%	37,9%	37,9%	45,9%
441	Information and reception employees (except in hotels)	9.003	14.647	4.884	0	9.762	68.634	13,1%	27,3%	27,3%	34,5%
442	Travel agency employees, hotel receptionists and telephonists	27.476	36.116	13.513	362	22.241	187.650	14,6%	26,5%	26,7%	33,9%
443	Opinion survey agents	994	798	450	0	348	10.267	9,7%	13,1%	13,1%	17,4%
444	Counter employees and similar (except ticket sellers)	9.480	15.573	6.138	0	9.435	71.695	13,2%	26,4%	26,4%	34,9%
450	Other clerical employees attending the public	91.709	192.546	66.307	0	126.238	566.165	16,2%	38,5%	38,5%	50,2%
500	Owner waiters and chefs	11.681	18.996	10.290	0	8.706	250.741	4,7%	8,1%	8,1%	12,2%
511	Waged chefs	5.128	20.345	13.788	0	6.557	259.267	2,0%	4,5%	4,5%	9,8%
512	Waged waiters	24.823	72.467	49.790	0	22.677	547.919	4,5%	8,7%	8,7%	17,8%
521	Section heads in shops and department stores	7.994	20.040	10.983	0	9.057	90.690	8,8%	18,8%	18,8%	30,9%
522	Salespersons in shops and department stores	34.998	153.052	84.574	0	68.479	758.070	4,6%	13,7%	13,7%	24,8%
530	Shopkeepers	19.546	60.386	23.039	0	37.348	391.917	5,0%	14,5%	14,5%	20,4%
541	Salespersons in kiosks and street markets	890	1.818	1.416	0	402	44.514	2,0%	2,9%	2,9%	6,1%

Sources: Afi, INE (EPA microdata)

Table 16. Workers who have completed maths-related studies by occupation and level of maths achieved (number and % total), 3/4

Classification of occupations according to CNO-11 (3 digits)		Workers according to the level of studies concerned						Determination of maths-intensive occupations according to the level of education completed (accum.)			
		High	Middle				Total	High	High+ upper vocational	High+ upper vocational +middle vocational	High+ upper vocational +middle vocational +secondary
			Total	Secondary	Middle vocational	Upper vocational					
Code	**Description**	**People**						**% total**			
542	Telemarketing operators	2.417	5.584	2.330	0	3.254	19.551	12,4%	29,0%	29,0%	40,9%
543	Petrol station attendants	594	11.050	4.883	0	6.167	37.639	1,6%	18,0%	18,0%	30,9%
549	Other salespersons	6.054	2.585	773	0	1.812	28.716	21,1%	27,4%	27,4%	30,1%
550	Cashiers and ticket sellers (except banks)	7.467	42.700	21.434	0	21.266	147.078	5,1%	19,5%	19,5%	34,1%
561	Nursing auxiliaries	9.399	206.661	158.734	0	47.927	279.911	3,4%	20,5%	20,5%	77,2%
562	Auxiliary Technicians in pharmacy and health emergencies and other carers of persons in health services							0,0%	0,0%	0,0%	0,0%
571	Home care workers (excluding childcare)	6.187	44.710	26.910	0	17.800	172.199	3,6%	13,9%	13,9%	29,6%
572	Child minders	4.490	16.960	11.274	0	5.686	119.710	3,8%	8,5%	8,5%	17,9%
581	Hairdressers and specialists in aesthetic and wellness treatments and similar	3.222	10.103	6.414	0	3.690	212.628	1,5%	3,3%	3,3%	6,3%
582	Workers attending to travellers, tourist guides and similar	3.037	6.116	2.563	0	3.553	44.684	6,8%	14,7%	14,7%	20,5%
583	Building maintenance and cleaning supervisors, concierges and butlers	7.225	19.993	7.565	0	12.429	107.799	6,7%	18,2%	18,2%	25,2%
584	Bed & breakfast keepers	673	1.009	220	0	789	6.360	10,6%	23,0%	23,0%	26,5%
589	Other personal service workers	136	6.282	3.936	0	2.346	29.854	0,5%	8,3%	8,3%	21,5%
591	Civil guards	1.946	8.334	3.876	0	4.458	52.165	3,7%	12,3%	12,3%	19,7%
592	Police officers	9.812	33.675	10.844	0	22.832	156.521	6,3%	20,9%	20,9%	27,8%
593	Fire-fighters	1.585	8.034	2.087	0	5.947	29.478	5,4%	25,6%	25,6%	32,6%
594	Private security personnel	6.268	29.891	17.436	0	12.455	134.147	4,7%	14,0%	14,0%	27,0%
599	Other workers in protection and security services	9.822	12.686	3.434	0	9.253	80.379	12,2%	23,7%	23,7%	28,0%
611	Qualified agricultural workers (except in horticulture, greenhouses, nurseries and gardens)	3.622	10.447	5.070	0	5.377	138.992	2,6%	6,5%	6,5%	10,1%
612	Qualified workers in horticulture, greenhouses, nurseries and gardens	3.165	18.381	13.257	319	4.805	127.295	2,5%	6,3%	6,5%	16,9%
620	Qualified livestock workers (including poultry, bees and similar)	2.304	9.966	4.565	0	5.401	110.941	2,1%	6,9%	6,9%	11,1%
630	Qualified workers in mixed farming	0	2.216	668	0	1.548	27.323	0,0%	5,7%	5,7%	8,1%
641	Qualified workers in forestry and the environment						4.229	0,0%	0,0%	0,0%	0,0%
642	Qualified workers in fishing and aquaculture	0	1.716	1.356	0	361	21.326	0,0%	1,7%	1,7%	8,0%
643	Qualified workers in hunting						185	0,0%	0,0%	0,0%	0,0%
711	Concrete, shuttering, steel-fixing workers and similar	0	1.436	1.436	0	0	22.288	0,0%	0,0%	0,0%	6,4%
712	Bricklayers, masons, and cutters, splitters and carvers of stone	1.533	35.440	16.506	0	18.933	259.786	0,6%	7,9%	7,9%	14,2%
713	Carpenters (excluding cabinetmakers and metal structure fitters)	3.781	16.458	11.008	0	5.449	110.178	3,4%	8,4%	8,4%	18,4%
719	Other workers on structural construction works	2.266	25.240	10.567	0	14.673	101.404	2,2%	16,7%	16,7%	27,1%
721	Plasterers and renderers	0	1.041	1.041	0	0	13.478	0,0%	0,0%	0,0%	7,7%
722	Plumbers and pipe fitters	191	12.762	8.856	0	3.906	64.375	0,3%	6,4%	6,4%	20,1%
723	Painters, wallpaperers and similar	0	18.494	10.317	0	8.178	104.516	0,0%	7,8%	7,8%	17,7%
724	Floor-layers, parquet fitters and similar	0	2.756	1.641	0	1.115	23.417	0,0%	4,8%	4,8%	11,8%
725	Refrigeration and air-conditioning fitters	0	15.124	7.263	1.139	6.722	34.969	0,0%	19,2%	22,5%	43,2%
729	Other finishing workers in construction, installations (except electricians) and similar	278	5.745	3.465	0	2.280	28.118	1,0%	9,1%	9,1%	21,4%
731	Moulders, welders, panel-beaters, metal structure fitters and similar	1.714	49.144	29.947	0	19.197	175.783	1,0%	11,9%	11,9%	28,9%
732	Blacksmiths, toolmakers and similar	551	43.170	19.722	0	23.448	102.764	0,5%	23,4%	23,4%	42,5%
740	Mechanics and machine adjusters	2.634	137.595	49.356	0	88.239	297.206	0,9%	30,6%	30,6%	47,2%
751	Electricians in construction and similar	258	71.166	36.441	1.264	33.462	128.902	0,2%	26,2%	27,1%	55,4%
752	Other installers and repairers of electrical equipment	2.138	71.596	34.564	0	37.032	104.503	2,0%	37,5%	37,5%	70,6%

Sources: Afi, INE (EPA microdata)

Table 17. Workers who have completed maths-related studies by occupation and level of maths achieved (number and % total), 4/4

Classification of occupations according to CNO-11 (3 digits)		High	Middle				Total	High	High+ upper vocational	High+ upper vocational +middle vocational	High+ upper vocational +middle vocational +secondary
			Total	Secondary	Middle vocational	Upper vocational					
Code	Description	People						% total			
753	Installers and repairers of electrical and telecommunications equipment	4.252	49.104	18.527	306	30.271	92.819	4,6%	37,2%	37,5%	57,5%
761	Metal precision mechanics, potters, glazers and artisans	720	6.205	2.616	0	3.590	39.425	1,8%	10,9%	10,9%	17,6%
762	Graphic arts workers	550	9.544	4.891	0	4.652	39.946	1,4%	13,0%	13,0%	25,3%
770	Workers in the food, drinks and tobacco industry	4.478	30.314	10.449	0	19.865	215.708	2,1%	11,3%	11,3%	16,1%
781	Wood treatment workers and similar	0	1.775	523	0	1.251	11.212	0,0%	11,2%	11,2%	15,8%
782	Cabinetmakers and similar	423	2.395	2.395	0	0	25.691	1,6%	1,6%	1,6%	11,0%
783	Textile, dressmaking, fur, leather and footwear workers	2.148	6.031	4.506	0	1.525	52.692	4,1%	7,0%	7,0%	15,5%
789	Gluers, divers, product testers and other workers and artisans various	0	1.583	410	0	1.173	9.535	0,0%	12,3%	12,3%	16,6%
811	Operators in mineral extraction and exploitation facilities	0	2.760	1.259	0	1.501	13.349	0,0%	11,2%	11,2%	20,7%
812	Operators in metal treatment facilities	955	16.776	7.386	0	9.390	60.026	1,6%	17,2%	17,2%	29,5%
813	Operators in facilities and machines for chemical, pharmaceutical and photosensitive materials	3.724	11.203	5.771	0	5.432	34.519	10,8%	26,5%	26,5%	43,2%
814	Operators in facilities and machines for wood treatment and processing, paper manufacture, and paper, rubber and plastic products							0,0%	0,0%	0,0%	0,0%
815	Operators of machines for the manufacture of textile, fur and leather products	0	2.550	1.746	0	803	41.674	0,0%	1,9%	1,9%	6,1%
816	Operators of machines for the manufacture of food, drink and tobacco products	0	9.074	4.788	0	4.286	40.839	0,0%	10,5%	10,5%	22,2%
817	Operators of laundry and dry-cleaning machines	0	3.894	3.231	0	663	33.575	0,0%	2,0%	2,0%	11,6%
819	Other operators of fixed facilities and machinery	708	19.061	9.860	0	9.201	72.713	1,0%	13,6%	13,6%	27,2%
820	Factory fitters and assemblers	2.626	26.410	11.910	0	14.501	101.135	2,6%	16,9%	16,9%	28,7%
831	Locomotive drivers and similar	414	3.401	652	0	2.748	11.051	3,7%	28,6%	28,6%	34,5%
832	Operators of mobile agricultural and forestry machinery	0	3.558	1.105	0	2.453	16.188	0,0%	15,2%	15,2%	22,0%
833	Operators of other mobile machinery	2.073	25.404	16.452	0	8.951	154.705	1,3%	7,1%	7,1%	17,8%
834	Seamen						9.073	0,0%	0,0%	0,0%	0,0%
841	Car, taxi and van drivers	1.345	56.602	33.293	0	23.309	256.969	0,5%	9,6%	9,6%	22,6%
842	Bus and tram drivers	880	16.923	9.043	0	7.881	71.110	1,2%	12,3%	12,3%	25,0%
843	Truck drivers	2.034	56.949	35.843	0	21.107	321.606	0,6%	7,2%	7,2%	18,3%
844	Motorcycle and moped riders	288	2.814	2.814	0	0	21.772	1,3%	1,3%	1,3%	14,2%
910	Domestic staff	14.747	32.931	20.453	0	12.478	427.391	3,5%	6,4%	6,4%	11,2%
921	Cleaners of offices, hotels, etc.	5.151	65.533	43.075	0	22.458	653.196	0,8%	4,2%	4,2%	10,8%
922	Cleaners of vehicles and windows and cleaning by hand	725	7.614	3.965	0	3.649	43.068	1,7%	10,2%	10,2%	19,4%
931	Kitchen assistants	3.813	9.891	8.271	0	1.620	145.602	2,6%	3,7%	3,7%	9,4%
932	Fast food preparers	0	1.359	0	0	1.359	8.283	0,0%	16,4%	16,4%	16,4%
941	Street vendors	0	659	659	0	0	4.501	0,0%	0,0%	0,0%	14,7%
942	Advertisement distributors, shoe shiners, and other street occupations						2.468	0,0%	0,0%	0,0%	0,0%
943	Office porter, luggage porter, messengers, etc.	3.425	7.639	3.963	0	3.676	39.146	8,7%	18,1%	18,1%	28,3%
944	Refuse collectors and sorters, street sweepers, etc.	0	16.188	9.930	0	6.257	99.395	0,0%	6,3%	6,3%	16,3%
949	Other basic occupations	1.413	7.715	3.924	0	3.790	29.206	4,8%	17,8%	17,8%	31,3%
951	Agricultural labourers	2.040	21.971	17.658	0	4.313	332.800	0,6%	1,9%	1,9%	7,2%
952	Livestock labourers	256	2.431	1.518	0	914	30.076	0,9%	3,9%	3,9%	8,9%
953	Mixed farming labourers	0	160	0	0	160	1.296	0,0%	12,4%	12,4%	12,4%
954	Fishing, aquaculture, forestry and hunting labourers	0	1.860	965	0	896	11.109	0,0%	8,1%	8,1%	16,7%
960	Construction and mining labourers	1.122	13.141	9.174	0	3.967	113.925	1,0%	4,5%	4,5%	12,5%
970	Manufacturing industry labourers	3.289	28.566	16.760	0	11.806	201.285	1,6%	7,5%	7,5%	15,8%
981	Transport labourers, unloaders, etc.	4.905	28.078	19.035	0	9.043	193.473	2,5%	7,2%	7,2%	17,0%
982	Shelf stackers	1.159	13.256	6.454	0	6.802	54.693	2,1%	14,6%	14,6%	26,4%
	Total	**2.865.522**	**3.200.279**	**1.405.715**	**7.668**	**1.786.896**	**17.850.492**				

Sources: Afi, INE (EPA microdata)

7. 4. PRODUCTS APPROACH

The aim of incorporating the products approach into the analysis is to improve the approximation to the importance of mathematics in GVA and employment provided by the maths-intensive occupations approach.

Mathematical language is present in a significant part of the stock of physical capital; in a good part of technology-intensive industrial activities and services, such as telecommunications services, it is largely responsible for the value of the output or final product. For example, a platform for intermediation in transportation services uses algorithms to organise efficiently user demands and service offerings. The incomes of workers without mathematical training in technological capital-intensive companies would not be visible under the occupations approach described above. However, the end products approach enables this undervaluation or natural limitation to be supplemented. This methodology was carried out in the following phases:

1. **Identification of technology-intensive sectors and assignation of the importance or mathematical intensity of the product:**

The first step is to apply a series of selection criteria. In particular, two:

i) Sectoral indicators: Productivity, share of gross operating surplus in total GVA of the sector and intensity of technological innovation.
ii) Literatura relevante y entrevistas a expertos con dedicación a la transferencia matemática.

The statistical sources used for this initial identification are the following:

- R&D Statistics in enterprises, compiled by the INE.
- Aggregates by sector in the national accounts, provided by the INE.
- Surveys to experts on the mathematical intensity of products.
- Mathematical intensity of products in the Australian economy.

2. **Comprehensive comparison at the sector level of the results of the occupations approach (EO) and products approach (EP).**

3. **Composition of the vector of integrated direct impact (EO and EP).**

7. 5. INDIRECT AND INDUCED IMPACTS

The methodology implemented to estimate the indirect and induced impacts on employment and GVA of mathematical research and mathematics technology transfer in the Spanish economy is that developed by the economist W. Leontieff[21]. This is widely used in sectoral analysis, due to its simplicity and the valuable information it provides, given the level of disaggregation by sector at which it works.

The essential idea behind this methodology is that each sector is linked to the others through trading in goods and services with the end purpose of supplying final demand of domestic and foreign households and

21 An American economist of Russian origin, who was honoured in 1973 with the Nobel Prize in Economics for "The development of the input-output method and its application to the most important economic problems". His analysis led to the introduction of matrix algebra for the treatment of the problems of general equilibrium.

other economic agents (public administrations, for example, or other companies demanding investment goods). This inescapable link between all the productive activities in an economy dictates that a few specific sectors are decisive in boosting the productive process or, on the contrary, hindering it. This is due to the degree of sophistication incorporated in their technology, their capacity for innovation and the nature of the goods and services that they produce.

The Input-Output Tables (IOT) are the starting point for measuring the impacts, and are defined as the statistical-accounting tool, with extensive sectoral breakdown. They show flows of production and distribution operations in a given country or region over a period of time between these customer-supplier sectors, as well as a sectoral breakdown of final demand and primary inputs (labour and capital). Its purpose is to provide a systematic description of the economy, its sectoral, functional and institutional components, and its relations with other economies. In Spain, the National Institute of Statistics (INE) is responsible for providing this information.

From a functional perspective, the IOTs analyse the economic activity of the economy's most significant productive sectors or aggregated branches of activity (usually between 50 and 100 large groups of productive activities), focusing on the process of intersectoral purchases and sales and the satisfaction of final demand, in addition to the consistency between total production (equivalent to the turnover of the productive units), income (or its equivalent, Gross Added Value) and employment.

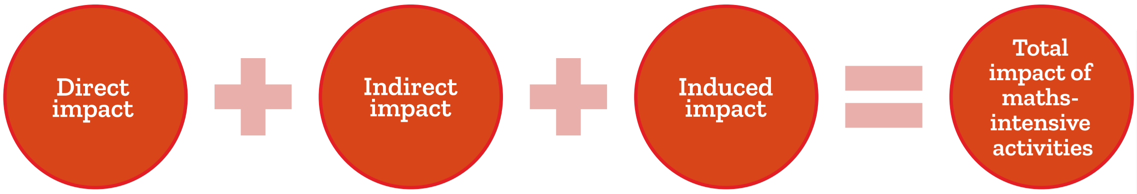
Diagram 8 summarises the list of direct, indirect and induced effects which together represent the total impact of the maths-intensive activities.

The **direct effects** are those produced by maths-intensive activities when they fulfil the final demand arising in the economy as a whole, or when they receive a demand shock or disturbance, experience the effects of an economic policy or simply undergo a structural change.

The **indirect effects** are those that occur in all other sectors as a result of the intermediate consumption by the maths-intensive activities that we are interested in studying.

The **induced effects** occur as a result of the spending of the income generated by the above effects on the economy as a whole. While the maths-intensive activities only exert indirect effects on those sectors closely linked to them, by demanding supplies, the induced effects, on the contrary, affect all sectors of the economy, depending on how the income generated by the above effects is distributed among the various components of final demand.

Diagram 8. Leontieff's methodology of indirect and induced impacts



Source: Afi

7. 6. ESTIMATED IMPACT WITH PANEL DATA METHODOLOGY

For the quantitative analysis, a panel database structure was used, composed of 66 countries during the 2013-2016 period. The variables used in the econometric estimation are shown in Table 18. Each of the three indicators (STEM students, STEM publications, PISA results) has been normalised as follows: for each of the years analysed (2013, 2014, 2015 and 2016), the country with the highest value was selected and given a value of 1. For the other countries, the distance to the maximum value was calculated. In other words, the value of the indicator for each country reflects its distance from the leading country. Each indicator takes values from 0 to 1. The mathematical human capital index is the sum of the three indicators, and takes values from 0 to 3. Then the following equation was calculated using a pooled OLS model:

$$Ln\,(GDP\ per\ worker)_{it} = \beta_0 + \beta_1\, ln\,(X)_{it} + \beta_2\, ln\,(capital\ stock\ by\ workers)_{it} + \beta_1\, Regions_i + \beta_n\, Year_t + \varepsilon_{it}$$

Where ***X*** represents, in turn, the mathematical human capital index, STEM students, academic publications in STEM disciplines and the PISA maths exam mark; ***i*** represents the country, ***t*** the year and ε_{it} the marginal error.

β0 = constant (average estimated GDP per worker when the rest of variables take value 0)
β1 = average estimated change in GDP per worker after a marginal change in variable X, ceteris paribus.
β2 = average estimated change in GDP per worker after a marginal change in capital stock by workers, ceteris paribus.
βj = average estimated difference in terms of GDP per worker between the “Mediterranean Europe” region and the rest of regions, ceteris paribus.
βn = average estimated difference in terms of GDP per worker between 2013 and the rest of Years, ceteris paribus.

Table 18. **Description of the variables used in the econometric analysis**

Variable	Medición	Fuente
STEM Students	Number of students in STEM branches as % of the total population	UNESCO
STEM Publications	Number of academic publications in STEM branches as % of the total population	SCIMAGO
PISA Results	Mark in the maths section of the PISA exam	OECD
GDP per worker	In constant dollars, PPP	World Bank
Capital Stock per worker	In constant dollars	World Bank

Source: Afi

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10. GLOSSARY

National Education Classification (CNED-14). The CNED-14 provides a system for classifying programmes by educational level and the level achieved. This dual classification has the single aim of ensuring that the statistical treatment of information on education and training is uniform, enabling the analysis and comparison of results between different statistical operations, and also between different countries, as it is compatible with the Normalised International Classification of Education (CINE-2011).

National Classification of Occupations (CNO-2011). The aim of the CNO-2011 is allow the uniform treatment of information on occupations for statistical purposes. It uses three-digit detail and has 170 different categories.

National Classification of Economic Activities (CNAE-2009). The CNAE-2009 is a system for classifying economic activities that can be used to provide national statistics that can be differentiated by the activities established and can classify statistical units and entities according to their economic activity. It uses three-digit detail and has 272 different categories.

Labour Force Survey (EPA). The EPA is a quarterly household survey which the INE has been carrying out since 1964. Its main aim is to collect data on the population regarding the labour market: employed, active, unemployed and inactive workers. The EPA surveys a sample of 65,000 families, some 200,000 individuals, every quarter. This report uses microdata for 2016 (the latest available with the required level of detail).

STEM skills. STEM is the acronym for Science, Technology, Engineering y Mathematics. This term was coined by the National Science Foundation (NSF) in the 1990s.

Direct impact. Estimated economic impact of a given sector or company on GDP and employment.

Indirect impact. Estimated economic impact on GVA and employment in other sectors of investment and spending on goods and services by a given sector or company in order to carry on its activity. It represents the knock-on effect on the supplier chain.

Induced impact. Estimated economic impact on GVA and employment resulting from spending the income (wages and company profits) generated by the above effects in the economy as a whole, depending on how this income is distributed among the different components of final demand.

Mathematical research and transfer (MRT). Acronym used in this report to define the activity of mathematical research and transfer of mathematical technology to the rest of the economy.

Full-time equivalent jobs (FTE). Full-time equivalent jobs (FTE) are calculated as total hours worked divided by the average annual hours worked in full-time jobs in a country (generally 40 hours per week). This measure is commonly used in Spain's National Accounts (INE) and differs from the metric used in the labour force survey (EPA).

GDP. Acronym for Gross Domestic Product. This measure is commonly used to quantify the income generated by a country over a given period of time (generally one year).

GDP per capita. GDP per capita or per capita income is an economic indicator measuring the relationship between a country's national income and its population. It is calculated by dividing the country's Gross Domestic Product (GDP) by the number of inhabitants.

PISA tests. The Programme for International Student Assessment (PISA) report is the result of a worldwide study by the Organisation for Economic Co-operation and Development (OECD) that measures school pupils' performance in mathematics, science and reading. Its aim is to provide comparable data, enabling countries to improve their educational policies and results.

Area of the level of studies and/or discipline. Codes established by the National Education Classification (CNED-2014) for each academic discipline, irrespective of their level (intermediate, university or higher).

Input-Output Table (TIO-2010). The Input-Output Table is a systematic representation of a country's economic activity, breaking down the national production between the sectors that produce it and those that use it. This technique was developed by Leontieff, who was awarded the Nobel Prize in Economics in 1973.

Table of origins. The table of origins of the input-output framework shows the production of goods and services, by product and type of supplier, of the different economic sectors and also the imports thereof.

Table of destinations. The table of destinations of the input-output framework shows the utilisation of goods and services, by product and type of utilisation (intermediate consumption by sector, final consumption, gross capital formation and exports). It also shows the components of Gross Domestic Product (GDP). TSIO-10. The classification of the symmetrical input-output table refers to the different types of sector for which the purchases and sales cross-referenced in the Input-Output Table are analysed. This report uses 63 different categories.

Gross Value Added (GVA). Gross Value Added (GVA) is the economic aggregate that measures the value added generated by all the producers in an economic area, reflecting the value added to goods and services in the different phases of production.

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